1. Consider the function
\[ f(x) = \sin \left( \frac{\pi}{2} x \right) + \frac{x}{2}. \]
(a) We would like to construct a polynomial \( P(x) \) that interpolates \( f(x) \) at the points \( x_0 = 0 \), \( x_1 = 2 \), and \( x_2 = 3 \). Compute all the necessary Lagrange polynomials and use these to construct the interpolating polynomial \( P(x) \).
(b) Compute the error term \( R(x) \).
(c) **Interpolation:** Compute the maximum bound on \( R(x) \) if we approximate \( f(1) \) by \( P(1) \).
(d) **Extrapolation:** Compute the maximum bound on \( R(x) \) if we approximate \( f(4) \) by \( P(4) \).

2. **SOURCE CODE:**

Let \( x_0, x_1, \ldots, x_n \) be distinct points with corresponding data values \( f_0, f_1, \ldots, f_n \).
Let \( P(x) \) be the unique polynomial of degree \( n \) which interpolates the data (i.e., \( P(x_i) = f_i \) for each \( i = 0, \ldots, n \)). Write the following MATLAB routines:

(a) \( F = \text{DivDiff}(x,f,n) \) — a function to compute the divided differences.
(b) \( P = \text{Horners}(x,F,xbar) \) — a function which uses the divided differences to evaluate \( P(x) \) at the point \( xbar \) using Horner’s method.
3. Consider the function

\[ f(x) = \frac{1}{1 + 25x^2}. \]

(a) Let \( x_i \) be 11 equally spaced points on \([-1, 1]\):

\[ x_i = -1 + i/5, \quad \text{where} \quad i = 0, 1, 2, \ldots, 10. \]

i. Use your code from the previous problem to construct the divided differences with these \( x_i \)'s.

ii. Use your Horner’s method code to evaluate the interpolating polynomial \( P(x) \) at all the points \( z_k \):

\[
\begin{align*}
  z &= \text{linspace}(-1,1,500); \\
  \text{for } k=1:500 \quad P(k) &= \text{Horners}(x,F,z(k)); \\
  \text{end}
\end{align*}
\]

iii. Make a single MATLAB plot that contains: \((z, P), (z, f(z))\), and the data points \((x_i, f(x_i))\) for \( i = 0, 1, 2, \ldots, 10 \). Make the \( P(z) \) plot a solid line ('b-'), the \( f(z) \) a dashed line ('r--'), and the data points stars ('k*').

(b) Now, let \( x_i \) be 11 unequally spaced points on \([-1, 1]\):

\[ x_i = \cos \left( \frac{\pi (i + 0.5)}{11} \right), \quad \text{where} \quad i = 0, 1, 2, \ldots, 10. \]

Repeat steps (i), (ii), and (iii) for these unequally spaced data points.

(c) Explain what you observe with these two choices of the \( x_i \)'s.