Assigned: Monday Oct. 31, 2011
Due Date: Friday, Nov. 11, 2011

MATH 174/274: Homework VI
"Polynomial Interpolation"
Fall 2011

1. Consider the function
$$f(x) = \sin\left(\frac{\pi}{2}x\right) + \frac{x}{2}\,.$$

   (a) We would like to construct a polynomial $P(x)$ that interpolates $f(x)$ at the points $x_0 = 0$, $x_1 = 2$, and $x_2 = 3$. Compute all the necessary Lagrange polynomials and use these to construct the interpolating polynomial $P(x)$.

   (b) Compute the error term $R(x)$.

   (c) **Interpolation:** Compute the maximum bound on $R(x)$ if we approximate $f(1)$ by $P(1)$.

   (d) **Extrapolation:** Compute the maximum bound on $R(x)$ if we approximate $f(4)$ by $P(4)$.

2. **SOURCE CODE:**

   Let $x_0, x_1, \ldots, x_n$ be distinct points with corresponding data values $f_0, f_1, \ldots, f_n$. Let $P(x)$ be the unique polynomial of degree n which interpolates the data (i.e., $P(x_i) = f_i$ for each $i = 0, \ldots, n$). Write the following MATLAB routines:

   (a) `F = DivDiff(x,f,n)` – a **function** to compute the divided differences.

   (b) `P = Horners(x,F,xbar)` – a **function** which uses the divided differences to evaluate $P(x)$ at the point `xbar` using Horner's method.

3. Consider the function

$$f(x) = \frac{1}{1 + 25x^2}\,.$$

(a) Let x_i be 11 equally spaced points on $[-1, 1]$:

$$x_i = -1 + i/5, \qquad \text{where} \quad i = 0, 1, 2, \ldots, 10.$$

i. Use your code from the previous problem to construct the divided differences with these x_i's.

ii. Use your Horner's method code to evaluate the interpolating polynomial $P(x)$ at all the points z_k:

```
z = linspace(-1,1,500);
for k=1:500
   P(k) = Horners(x,F,z(k));
end
```

iii. Make a single MATLAB plot that contains: (z, P), $(z, f(z))$, and the data points $(x_i, f(x_i))$ for $i = 0, 1, 2, \ldots 10$. Make the $P(z)$ plot a solid line (`'b-'`), the $f(z)$ a dashed line (`'r--'`), and the data points stars (`'k*'`).

(b) Now, let x_i be 11 *unequally* spaced points on $[-1, 1]$:

$$x_i = \cos\left[\frac{\pi\,(i + 0.5)}{11}\right], \qquad \text{where} \quad i = 0, 1, 2, \ldots, 10.$$

Repeat steps (i), (ii), and (iii) for these unequally spaced data points.

(c) Explain what you observe with these two choices of the x_i's.