# MATH 270C ASSIGNMENT 2

(1) Study, as functions of $\alpha \in \mathbb{R}$, stability and order of the family of multistep methods,

$$u_{n+1} = \alpha u_n + (1-\alpha)u_{n-1} + 2hf_n + \frac{h\alpha}{2}[f_{n-1} - 3f_n]$$

(2) Adams methods can be easily generalized, integrating between $t_{n-r}$ and $t_{n+1}$ with $r \geq 1$. Show that, by doing so, we get methods of the form

$$u_{n+1} = u_{n-r} + h \sum_{j=-r}^{p} b_j f_{n+j},$$

where $p = 1$ for an implicit method, and $p = 0$ for an explicit method.

Show that for $r = 1$, $p = 0$, the midpoint method, $u_{n+1} = u_{n-1} + 2hf_n$, is recovered (the methods of this family are called *Nyström methods*).

(3) Show that the explicit multistep method

$$y_{n+3} + \alpha_2 y_{n+2} + \alpha_1 y_{n+1} + \alpha_0 y_n = h[\beta_2 f(t_{n+2}, y_{n+2}) + \beta_1 f(t_{n+1}, y_{n+1}) + \beta_0 f(t_n, y_n)]$$

is fourth-order only if $\alpha_0 + \alpha_2 = 8$ and $\alpha_1 = -9$. Hence deduce that this method cannot be both fourth-order and convergent.