(1) (Iserles, 4.1) Let \( y' = \Lambda y \), \( y(t_0) = y_0 \), be solved (with a constant step size \( h > 0 \)) by a one-step method with a function \( r \) that obeys the relationship \( y_n = [r(h\lambda)]^n \) (when applied to the scalar model equation). Suppose that a nonsingular matrix \( V \) and a diagonal matrix \( D \) exist such that \( \Lambda = VDV^{-1} \). Prove that there exist vectors \( x_1, x_2, \ldots, x_d \in \mathbb{R}^d \) such that

\[
y(t_n) = \sum_{j=1}^{d} e^{t_n\lambda_j} x_j, \quad n = 0, 1, \ldots,
\]

and

\[
y_n = \sum_{j=1}^{d} [r(h\lambda_j)]^n x_j, \quad n = 0, 1, \ldots,
\]

where \( \lambda_1, \ldots, \lambda_d \) are the eigenvalues of \( \Lambda \). Deduce that the values of \( x_1 \) and \( x_2 \) in (4.3) and (4.4) are identical.

(2) (Iserles, 4.6) Evaluate explicitly the function \( r \) for the following Runge–Kutta methods:

(a)

\[
\begin{array}{c|ccc}
0 & 0 & 0 \\
\frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\
\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\
\end{array}
\]

(b)

\[
\begin{array}{c|ccc}
\frac{1}{3} & \frac{1}{3} & 0 \\
\frac{1}{2} & \frac{3}{4} & \frac{1}{4} \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\
\end{array}
\]

(c)

\[
\begin{array}{c|cccc}
0 & 0 & 0 & 0 \\
\frac{2}{3} & \frac{1}{2} & \frac{1}{2} & 0 \\
\frac{1}{6} & \frac{1}{3} & \frac{1}{3} & 0 \\
\frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\
\end{array}
\]

Are these methods A-stable?

(3) (Iserles, 4.8) Determine the order of the two-step method

\[
y_{n+2} - y_n = \frac{2}{3} h \left[ f(t_{n+2}, y_{n+2}) + f(t_{n+1}, y_{n+1}) + f(t_n, y_n) \right], \quad n = 0, 1, \ldots
\]

Is it A-stable?