# MT2 Solns



1) i) No  ii) Yes  iii) 3  iv) 1  v) 3  vi) No  vii) Yes

viii) $|Z|$  ix) $\det(A)^{-1}$  x) 1.

2) a) $\det A = 1\begin{vmatrix} -1 & 6 \\ 2 & 1 \end{vmatrix} + 2\begin{vmatrix} -1 & -1 \\ 4 & 2 \end{vmatrix} = (-1-12) + 2(-2+4) = \boxed{-9}$

b) $\det B = 3i(3+2i) - 6i = \boxed{-6+3i}$

c) $\det C = \boxed{1}$ switching $R_1 \leftrightarrow R_2$, $R_3 \leftrightarrow R_5$ makes $C$ $I_5$

d) $\det D = \boxed{0}$ since $R_1$ & $R_3$ are equal.

3) a) $\operatorname{adj}(A) = \begin{pmatrix} \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} & -\begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} & \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} \\ -\begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} \end{pmatrix} = \boxed{\begin{pmatrix} 2 & -4 & 2 \\ -2 & 2 & -1 \\ 0 & 0 & -1 \end{pmatrix}}$

b) $\det(A) = 2\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = 2(1-2) = \boxed{-2}$

c) $A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A) = \begin{pmatrix} 2/-2 & -4/-2 & 2/-2 \\ -2/-2 & 2/-2 & -1/-2 \\ 0 & 0 & -1/-2 \end{pmatrix} = \boxed{\begin{pmatrix} -1 & 2 & -1 \\ 1 & -1 & 1/2 \\ 0 & 0 & 1/2 \end{pmatrix}}$

d) $A\vec{x} = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} \iff \vec{x} = A^{-1}\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$

$\Rightarrow \vec{x} = \begin{pmatrix} -1 & 2 & -1 \\ 1 & -1 & 1/2 \\ 0 & 0 & 1/2 \end{pmatrix}\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 + \cancel{2} - \cancel{2} \\ -1 - \cancel{1} + \cancel{1} \\ 0 + 0 + 1 \end{pmatrix} = \boxed{\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}}$

4) $\pi = (1\ 2\ 4\ 5)$ in disjoint cycle notation.

a) Since $(a_1\, a_2 \cdots a_k) = (a_1\, a_k)(a_1\, a_{k-1}) \cdots (a_1\, a_2)$,

$$\boxed{\pi = (15)(14)(12)}$$

b) $\text{sign}(\pi) = \boxed{-1}$, since it can be written as an odd # (3) of permutations.

c) Note, for $i<j$, $(i\,j) = (i\ i{+}1)(i{+}1\ i{+}2)\cdots(j{-}1\ j)(j{-}2\ j{-}1)\cdots(i\ i{+}1)$

so $(15) = (1\,2)(23)(34)(45)(34)(23)(12)$

$(14) = (12)(23)(34)(23)(12)$

$(12)$ already an adjacent transposition.

$$\Rightarrow \pi = (15)(14)(12)$$

$$= \underbrace{(12)(23)(34)(45)(34)(23)(12)}_{(15)}\underbrace{(12)(23)(34)(23)(12)}_{(14)}(12)$$

$$= \boxed{(12)(23)(34)(45)(23)(12)}$$

since transpositions are their own inverses

5) a) $\det A = -2\begin{vmatrix} -1 & -1 \\ 3 & 2 \end{vmatrix} - 1\begin{vmatrix} 4 & -1 \\ 2 & 3 \end{vmatrix} = -2(-2+3) - 1(12+2)$

$= -2 - 14 = \boxed{-16}$

b) check: $\det A_1 = -7$

$\det A_2 = 6$

$\det A_3 = -2$

c) By Cramer's Rule, $\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$, where $x_i = \dfrac{\det(A_i)}{\det(A)}$

$$= \begin{pmatrix} -7/-16 \\ 6/-16 \\ -2/-16 \end{pmatrix} = \boxed{\begin{pmatrix} 7/16 \\ -3/8 \\ 1/8 \end{pmatrix}}$$

6) See HW 8 #2 solns.