Please check the website for your room assignment!

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General remarks:

- You will be asked to write down your name and student ID on the front page.
- You have 50 minutes to work on the exam.
- No books, calculators, phones, or cheatsheets are allowed during class.
- The first to problems will take the form of a quiz: you have to answer true-false questions.
- Homework 1, Exercise 2, 4, 5
- Homework 2, Exercise 1, 2
- Homework 3, Exercise 1, 2, 3, 4
- It is assumed that you can handle basic matrix and vector algebra and that you have mastered all the elementary definitions of the course.

Problem 1.
Consider the matrix $A \in \mathbb{R}^{3\times 3}$ and the vector $b \in \mathbb{R}^3$ given by

$$A = \begin{pmatrix} 2 & 6 & 2 \\ -3 & -8 & 0 \\ 4 & 5 & 2 \end{pmatrix}, \quad b = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}.$$

(1) Write down an upper triangular matrix B that is row equivalent to A.
(2) Factorize $A = LU$, i.e., write A as a product, where $L \in \mathbb{R}^{3\times 3}$ is lower triangular and $U \in \mathbb{R}^{3\times 3}$ is upper triangular.
(3) Solve the linear system of equations $Ax = b$.

Solution 1. (1)

$$\begin{pmatrix} 2 & 6 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 19 \end{pmatrix}$$

(2)

$$\begin{pmatrix} 2 & 6 & 2 \\ -3 & -8 & 0 \\ 4 & 5 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -\frac{3}{2} & 1 & 0 \\ 2 & -7 & 1 \end{pmatrix} \begin{pmatrix} 2 & 6 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 19 \end{pmatrix}$$

(3)

$$x = \begin{pmatrix} 3/19 \\ -13/19 \\ 91/38 \end{pmatrix}$$

Problem 2.

(1) Write down an elementary matrix in $\mathbb{R}^{3\times 3}$ which, when applied to a vector $v \in \mathbb{R}^3$ through matrix–vector multiplication, exchanges the first and second coordinate of the vector v.
(2) Factorize the following matrix into elementary matrices

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{pmatrix}$$

Solution 2.

(1)

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(2)

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$