MATH 109 – HOMEWORK 2

Due Friday 26th. Handwritten submissions only.
The exercises in this homework are worth 16 points.

Exercise 1
A logical proposition that is composed from statements $A, B, C, \ldots$ through a combination of negation, disjunction, and conjunction is called in *conjunctive normal form* if it is the conjunction of disjunctions of terms from $A, \neg A, B, \neg B, \ldots$.

For example, the following proposition is in conjunctive normal form:

$$(C \vee \neg A \vee \neg B) \wedge (\neg B \vee A) \wedge (\neg C \vee B \vee A).$$

For each of the following propositions, find an equivalent proposition in conjunctive normal form:

- $((A \vee B) \wedge \neg B) \vee (B \wedge \neg(A \vee C) \wedge \neg(A \vee B)) \vee (A \wedge B)$
- $(\neg B \wedge A) \vee (\neg A) \vee (\neg C \wedge \neg(A \vee B))$
- $\neg((A \vee \neg C) \wedge (C \wedge B) \wedge \neg(A \vee B \vee \neg C))$

Exercise 2
Assume that we have parametrized statements $X(a, b)$ and $Y(a)$ that satisfy

$$X(a, b) \iff Y(a) \wedge Y(b).$$

Show that the following equivalence holds:

$$(X(a, b) \wedge Y(c)) \iff \neg(\neg Y(a) \vee \neg X(b, c)).$$

Exercise 3
Let x, y, z be three irrational numbers. Show that there are two of them whose sum is again irrational.

Exercise 4
Find all the pairs of non-zero real numbers (x, y) which satisfy

$$x + \frac{x}{y} = \frac{8}{3}, \qquad y + \frac{1}{x} = \frac{5}{2}.$$

Exercise 5
Find all real numbers x that satisfy the equation

$$8^x + 2 = 4^x + 2^{x+1}.$$

Exercise 6
Let a be an odd number. Show that there exists an integer k such that $a^2 = 8k + 1$.