

MATH 109 – HOMEWORK 3

*Due Friday, February 2nd. Handwritten submissions only.
The exercises in this homework are worth 16 points.*

Exercise 1

Let A , B , and C be sets. Prove the following statements:

- $(A \setminus B) \setminus C = A \setminus (B \cup C)$
- $A \setminus (B \setminus C) = (A \setminus B) \cup (A \cap C)$
- $(A \cap B) \setminus C = (A \setminus C) \cap (B \setminus C)$
- $(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C)$
- $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$
- $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$

Hint: Use the correspondence between elementary set operations and logical connectives.

Exercise 2

For each of the sums

$$A_n := \sum_{k=1}^n k, \quad B_n := \sum_{k=1}^n k^5,$$

determine the natural numbers $n \in \mathbb{N}$ for which the respective sum is even. Prove your result.

Exercise 3

Let x and y be real numbers. Prove the two inequalities

$$|x + y| \leq |x| + |y|, \quad |x - y| \geq ||x| - |y||.$$

Exercise 4

Let A , B , C , and D be sets with ten elements each, and suppose that the intersections of two of each have at least nine elements.

- (1) Show that the intersection $A \cap B \cap C$ is non-empty.
- (2) Show that the intersection $A \cap B \cap C \cap D$ is non-empty.