MATH 109 – HOMEWORK 8

Due Friday, March 8st. Handwritten submissions only.
The exercises in this homework are worth 16 points.

Problem 1
Consider the two functions

$$f : \mathbb{R} \setminus \{1,2,3\} \to \mathbb{R}, \quad x \mapsto \frac{5x^3 - 8x^2 - 27x + 18}{(x-1)(x-2)(x-3)},$$
$$g : \mathbb{R} \setminus \{0,1,2\} \to \mathbb{R}, \quad x \mapsto \frac{5x^3 + 7x^2 - 6x}{x(x-1)(x-2)},$$

Find the largest set $A \subseteq \mathbb{R}$ such that $A \subseteq \operatorname{dom}(f)$ and $A \subseteq \operatorname{dom}(g)$ and $f_{|A} = g_{|A}$.

Problem 2
Let X be a set. Consider the following relation $\sim$ on the power set $\mathfrak{P}(X)$:

$$\forall A, B \in \mathfrak{P}(X) : A \sim B \leftrightarrow A \cap B \neq \emptyset$$

Is $\sim$ reflexive, symmetric, or transitive? Either prove or give a counterexample.

Problem 3
Let R be a binary relation over a set X that is both symmetric and antisymmetric. Prove that R is a subset of the equality relation over X.

Problem 4
We introduce the following relation R on the rational numbers $\mathbb{Q}$. For $x, y \in \mathbb{Q}$ we write $x \sim_R y$ if there exist natural numbers $m, n \in \mathbb{N}$ such that $x^m = y^n$. Show that R is an equivalence relation.

Problem 5
Consider the following relation on the real numbers: for all $x, y \in \mathbb{R}$ we write

$$x \sim y \qquad :\Longleftrightarrow \qquad |x| \leq |y|.$$

Show that $\sim$ is reflexive and transitive, but not antisymmetric.

Problem 6
Let X be a set. Under which conditions is the empty set $\emptyset$ a partial order over X?

Problem 7 (Ungraded)
Consider the set $\mathbb{R}$ equipped with the canonical order $\leq$. Let ∞ and $-\infty$ be two symbols and define

$$\overline{\mathbb{R}} := \{-\infty, \infty\} \cup \mathbb{R}.$$

We define a relation R on $\overline{\mathbb{R}}$ as follows:

$$\forall x, y \in \mathbb{R} : (\quad x \sim_R y \quad :\Longleftrightarrow \quad x \leq y \quad),$$
$$\forall x \in \overline{\mathbb{R}} : (-\infty \leq x) \wedge (x \leq \infty),$$
$$-\infty \leq -\infty, \qquad -\infty \leq \infty, \qquad \infty \leq \infty.$$

Show that R is a partial order over $\overline{\mathbb{R}}$.

Problem 8 (Ungraded)
Let R denote the canonical order of set $N_0^8 = \{0, 1, \ldots, 7, 8\}$. Suppose that S is another partial order over N_0^8 such that $R \subseteq S$ and $(8, 0) \in S$. Show that S is the equality relation over N_0^8.