Part 2: Set Theory

Ordered Pairs and Cartesian Products

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Ordered Pairs: Definition

For two things x and y we let

$$(x, y)$$

denote the **ordered pair** with first entry x and second entry y.

The order of the entries of an ordered pair matters!

Two ordered pairs are the same if and only if corresponding entries are the same

$$(a, b) = (x, y) \iff a = x \wedge b = y.$$

For the time being, we treat ordered pairs as “language primitives”. Later we consider a trick how to construct ordered pairs in a purely set-theoretical way.

Cartesian Product: Let A and B be sets. The **Cartesian Product** of A and B is denoted by $A \times B$ and defined by

$$A \times B := \{(a, b) \mid a \in A \wedge b \in B\}.$$

We have

$$(a, b) \in A \times B \iff a \in A \wedge b \in B.$$

Does there exist a set with these properties?

Ordered Pairs: Set-theoretical Definition by Kuratowski

For two things x and y we define

$$(x, y) := \big\{\{x\}, \{x, y\}\big\}$$

We call this the **ordered pair** whose first component is x and whose second component is y.

Ordered Pairs: Set-theoretical Definition by Kuratowski

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We call this the **ordered pair** whose first component is x and whose second component is y.

For two sets A and B we define

$$\begin{aligned} A \times B &:= \left\{ \, \mathbf{p} \in \mathfrak{P}(\mathfrak{P}(A \cup B)) \;\middle|\; \exists a \in A, b \in B : \mathbf{p} = (a, b) \, \right\} \\ &= \left\{ \, \mathbf{p} \in \mathfrak{P}(\mathfrak{P}(A \cup B)) \;\middle|\; \exists a \in A, b \in B : \mathbf{p} = \{\{a\}, \{a, b\}\} \, \right\}. \end{aligned}$$

In particular, we have $A \times B \subset \mathfrak{P}(\mathfrak{P}(A \cup B))$.

Questions?