# Mathematical Reasoning: Introduction

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# Computation is Reasoning

Consider the following two linear equations:

$$2x + 2y = 1, \tag{1}$$

$$4x + 3y = 4. \tag{2}$$

we seek for the solution $(x, y)$.

(i) Both equations are statements about numbers, including the yet unknown numbers $x$ and $y$.

(ii) Using the first equation (1) we get

$$2 = 2 \cdot 1 = 2(2x + 2y) = 4x + 4y. \tag{3}$$

(iii) We use (1), (2), and the known fact $4 - 2 = 2$ to derive

$$2 = 4 - 2 = (4x + 3y) - (4x + 4y) = -y$$

But then $-2 = -(-y) = y$ follows.

Consider the following two linear equations:

$$2x + 2y = 1, \tag{4}$$

$$4x + 3y = 4. \tag{5}$$

we seek for the solution $(x, y)$.
(iv) We now know that $y = -2$. Now (4) implies

$$1 = 2x + 2y = 2x + 2 \cdot (-2) = 2x - 4.$$

(v) Using $1 = 2x - 4$, we get

$$5 = 1 + 4 = 2x - 4 + 4 = 2x.$$

But then

$$\frac{5}{2} = \frac{2x}{2} = x.$$

We conclude $x = 5/2$.

This was Gaussian elimination for a linear system of two equations in two variables.

Gaussian elimination can be seen as a computational procedure to solve a linear system of equations.

The steps in Gaussian elimination correspond to logical reasoning. We could have dissected each step much more into the details.

When computing by hand, it sometimes helps to forget the computation scheme and use the equations "directly".

Deep down, even $1 + 1 = 2$ can be rewritten as a logical conclusion from first axioms.

# Proofs in Mathematics

A proof is a text about mathematical objects that from first assumptions (**premises**) leads to **conclusions**.

**A mathematical text is first and foremost a text.**

Mathematical symbols are extension of the human language script.

A proof is correct if it makes sense. **That simple.**

A proof is preferably readable too.

A mathematical text (in a book) can be given as prose with mathematical symbols. Often, however, it is structured to make things more readable.

- Theorem, Lemma, Proposition, Corollary
- Definition
- Example
- Remark

# Connections beyond Mathematics

Mathematical reasoning in relation to

- Language
- Physics
- Computer Science
- Philosophy
- ...

## Language

Formal mathematics is a language.

*Hence this is a language course.* You have to practice to become fluent in the language of mathematics.

This is **not** about learning content that you will never use again. This is about the stuff that you will use in any other mathematics class.

## Philosophy

Logic is the border regions between mathematics and philosophy.

Both disciplines are (supposedly) based on logical reasoning.

Foundational theory of mathematics and logic has been subject of study for a long time.


- Willard Van Orman Quine. *Elementary logic*. 1941. Harvard University Press.
- https://en.wikipedia.org/wiki/Logic
- https://en.wikipedia.org/wiki/Mathematical_Logic
- https://en.wikipedia.org/wiki/Foundations_of_mathematics

## Computer Science

The design of algorithms and programming languages requires mathematical reasoning.

The design of software libraries requires mathematical reasoning.

Large parts of mathematics are concerned with theoretical computer science.

Donald Knuth. *The Art of Computer Programming, Vol. I - VII*. 1968-ongoing. Addison-Wesley.

Homotopy Type Theory: Univalent Foundations of Mathematics. `https://homotopytypetheory.org/book/`

**Physics**

Physics uses mathematics as a tool to formulate theories, assuming that phenomena can be described in mathematics.

Axiomatic approach shared with mathematics: start from foundational principles (physical laws) to derive descriptions of physical settings.

Theoretical physics uses physical intuition to derive formulas (*less formally than mathematics*) and as a golden threat (*often helpful for mathematicians*).