Part 4: Functions

Tuples and Sequences

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We can use functions to *implement* certain mathematical concepts in a neat manner.

- Multivariate tuples
- Sequences

Implementing Tuples

Tuples generalize ordered pairs ("duples").

Tuples are ordered sequences of things. We let X^n denote the n-tuples of members of X:

$$(x_1, \ldots, x_n) \in X^n.$$

We also write

$$X^n = \underbrace{X \times \cdots \times X}_{n-\text{times}}.$$

We can identify the set X^n with the set of functions from $\{1, \ldots, n\}$ into the set X.

Remark: We can use the definition of ordered pair to define $X \times X$. Using functions covertly, we can define $X \times X \times X$, $X \times X \times X \times X$, etc.

Implementing Tuples

Givens sets $X_1, \ldots, X_n$, we write $X_1 \times \cdots \times X_n$ for the set of ordered n-tuples whose i-th component is a member of X.

$$(x_1, \ldots, x_n) \in X_1 \times \cdots \times X_n.$$

We can identify the set $X_1 \times \cdots \times X_n$ with the set of functions from $\{1, \ldots, n\}$ into the union $X_1 \cup \cdots \cup X_n$ which map $i \in \{1, \ldots, n\}$ to a member of X_i.

Examples:

- $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$
- $\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R}$
- $\mathbb{R}^n = \underbrace{\mathbb{R} \times \cdots \times \mathbb{R}}_{n-\text{times}}$
- $\mathbb{R} \times \mathbb{Z} \times \mathbb{Z}$
- $\mathbb{Z} \times \mathbb{R} \times \mathbb{Z}$
- $\mathbb{Q}^3$

Implementing Sequences

Given a set X, a sequence in X is a function $x : \mathbb{N} \to X$ from the natural numbers to the set X.

$$x_1, x_2, \ldots$$

Instead of $\mathbb{N}$, the index set can also be $\mathbb{N}_0$.