Math 4C Summer I 2018 - Midterm Exam

Instructions: No calculators or electronic devices are allowed. Turn off and put away your cell phone. You may not use any notes, books, or resources. Make sure your solutions are clear and legible, and show all your work. Credit may not be given for unreadable or unsupported answers.

Question 0. (1 point) Carefully read all instructions above and make sure you have followed them all, as well as any other instructions given by your instructor during the exam.

Question 1. (10 points) Let $\ell$ be the line through the point (2, -1) with slope $-\frac{1}{3}$.

a) Find the equation of the line $\ell$. Simplify your answer so that it is in $y = mx + b$ form.

\[
y = -\frac{1}{3}x + b
\]

\[-1 = -\frac{1}{3}(2) + b
\]

\[b = -\frac{1}{3}
\]

$\Rightarrow$

\[
y = -\frac{1}{3}x - \frac{1}{3}
\]

b) Find the equation of the line through (3, 4) that is perpendicular to $\ell$.

\[
slope\ of\ the\ line\ :\ 3
\]

\[
y = 3x + b
\]

\[4 = 9 + b
\]

\[b = -5
\]

\[
y = 3x - 5
\]
Question 2. (10 points) Let \( f(x) = \log_4(3x + 1) \).

a. Find the domain of \( f(x) \).

\[
3x + 1 > 0 \\
x > -\frac{1}{3}
\]

b. Find a formula for \( f^{-1}(y) \).

\[
\log_4(3x + 1) = y \\
4^y = 3x + 1 \\
4^y - 1 = 3x \\
\Rightarrow \frac{4^y - 1}{3} = x
\]

\[
\Rightarrow f^{-1}(y) = \frac{y}{3} - \frac{1}{3}
\]
Question 3. (30 points) Solve for $x$.

a. $-2|4x + 2| - 15 \leq 5.$

\[
-2 |4x + 2| \leq 20
\]
\[
|4x + 2| \geq -10
\]
\[
\begin{cases}
4x + 2 \geq -10 \\
4x + 2 \leq +10
\end{cases} \Rightarrow 4x \geq -12 \quad \Rightarrow \quad x \geq -3
\]
\[
4x \leq 8 \Rightarrow x \leq 2
\]

b. $\log(4x) + \log(x + 1) = \log(2x - 1)^2$.

\[
\log 4x(x+1) = \log (2x-1)^2
\]
\[
\Rightarrow 4x^2 + 4x = 4x^2 - 4x + 1
\]
\[
-8x = 1 \quad \Rightarrow \quad x = \frac{1}{8}
\]

c. $x - 5\sqrt{x} + 4 = 0$

\[
\sqrt{x} = y \quad y^2 - 5y + 4 = 0
\]
\[
y_1, y_2 = \frac{5 \pm \sqrt{25 - 16}}{2} = \frac{5 \pm 3}{2} < y_1 = 4 \quad y_2 = 1
\]
\[
\Rightarrow \begin{cases}
\sqrt{x} = 4 \quad \Rightarrow \quad x = 16 \\
\sqrt{x} = 1 \quad \Rightarrow \quad x = 1
\end{cases}
\]
Question 4. (10 points) Find a number $t$ such that the equation $tx^2 + 5x + 4 = 0$ has no solution.

\[ b^2 - 4ac < 0 \]

\[ 25 - 4(t)(4) < 0 \]

\[ 25 - 16t < 0 \]

\[ \frac{25}{16} < t \]

Any $t > \frac{25}{16}$ works!
Question 5. (5 points) Simplify the expression $\left(\frac{(x^3y^7)^2}{x^4y^{-1}}\right)^2$.

\[
\left(\frac{-6x^4y^{12}}{4x^4y^{-1}}\right)^2 = \left(\frac{-12x^8y^{13}}{-8x^4y^{-2}}\right) = \frac{18y^{18}}{20x}
\]
Question 6. (15 points) Let \( f(x) = x^3 + 1 \) and \( g(x) = 2x^5 + x^2 \).

a. How many zeros could \( g(x) \) have?

\[ \text{at most 5}. \]

b. Describe the end behavior of \( \frac{f(x)}{g(x)} \).

\[
\frac{f(x)}{g(x)} = \frac{x^3 + 1}{2x^5 + x^2} = \frac{x^3}{2x^5} = \frac{1}{2x^2}
\]

\[
x \to \infty \Rightarrow \frac{f(x)}{g(x)} \to 0
\]

\[
x \to -\infty \Rightarrow \frac{f(x)}{g(x)} \to 0
\]

c. Describe the end behavior of \( g(x) \).

\[
x \to \infty \Rightarrow g(x) \to \infty
\]

\[
x \to -\infty \Rightarrow g(x) \to -\infty
\]
Question 7. (20 points) Fill in the blank:

a. Let \( f(x) \) be a function and \( x \) be in the domain of \( f \). Then \( f \circ f^{-1}(x) = \cdots x \cdots \).

b. The equation of the graph \( g(x) \) that is obtained by horizontally stretching the graph of \( f(x) \) 8 units and by shifting down 2 units is \( \cdots \). 

\[ g(x) = f\left(\frac{x}{8}\right) - 2 \]

c. The degree of the polynomial \( f(x) = x^2 + x^5 + 2^{10} + 20 \) is \( \cdots 10 \). 

d. An example of a polynomial of degree three whose only zeros are \(-5, 5\) is \( \cdots \).

\[ f(x) = (x + 5)(x - 5)^2 \quad \text{or} \quad f(x) = (x + 5)^2(x - 5) \]

e. The function \( g(x) = 12x^2 + |x| \) is a function that is even, odd, or neither \( \cdots \) even.

\[ g(-x) = 12(-x)^2 + |-x| = 12x^2 + |x| = g(x) \]

\[ \implies \text{even} \]
Question 8. (9 points) Extra Credit. Does the equality \( \frac{1 + \log_2 x}{2} = \log_2 \sqrt{2x} \) hold? Justify your answer.

Yes!

left hand side: \( \log_2 \sqrt{2x} = \log_2 (2x)^{\frac{1}{2}} = \frac{1}{2} \left( \log_2 (2x) \right) = \frac{1}{2} \left( \log_2 2 + \log_2 x \right) = \frac{1}{2} \left( 1 + \log_2 x \right) = \frac{1 + \log_2 x}{2} = \text{right hand side.} \)