1 Practice questions for midterm 1

1: Let $x \in A \cup B$ but $x \notin C$. If $x \in A$, we are done. Suppose then that $x \in B$. Since $x \notin C$, in fact we have that $x \in B \setminus C$, and therefore we are again done. Equality fails with the following sets: $A = \{1, 2\}$, $B = \{1, 3\}$ and $C = \{2, 3\}$. In fact, $(A \cup B) \setminus C = \{1\}$, but $A \cup (B \setminus C) = \{1, 2\}$.

2: Suppose that $7 \mid n$. Then $7 \mid n \cdot n^2$ and therefore $7 \mid n^3$, contradiction.

3: Yes, if $Q$ is true, then $P \implies Q$ is always true. If $Q$ is false, then it does not matter what $P \implies Q$ is.

4: Suppose that there is, and let’s call it $n$. Then pick $m = n + 1$, which is an integer. Since $m > n$ we get a contradiction.

5: Induction on $n$. Base case $n = 1$: $2 - 1 = 1 = 1^2$. Inductive step:

$$\sum_{i=1}^{n+1} (2i - 1) = \sum_{i=1}^{n} (2i - 1) + 2(n + 1) - 1 = n^2 + 2n + 1 = (n + 1)^2$$

6: Suppose that $0 \mid a$. Then there exists $b$ such that $a = 0 \cdot b = 0$, therefore $a = 0$. On the other hand, if $a = 0$, then $0 = 0 \cdot 0$ shows that $0 \mid a$. 