

# A GLOSSARY OF FINANCIAL TERMS

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## 1. INTRODUCTION

This document lays out some of the basic definitions of terms used in financial markets. First of all, the main types of financial entities that we shall consider are (i) **stocks**, issued by companies to raise money by offering a share of the company to shareholders, who then own the company; (ii) **bonds**, issued by companies, governments and other organizations to raise money by paying a fixed rate of interest for a fixed number of years, at which time the principle is paid back in full; (iii) **currency**, issued by countries, whose value relative to that of other countries may fluctuate; (iv) **futures and options**, which are financial instruments whose value is determined by the value of some **basic equity**, such as (i), (ii) or (iii). The term **financial derivative** or **contingent claim** is also used for (iv), and the equity on which it is based is called the **underlying equity**.

Each of (i), (ii), (iii) and (iv) are traded in specialized exchanges. For example, the NY Stock Exchange is a center for trading certain stocks, and the Chicago Mercantile Exchange trades in a wide range of futures and options.

**CAUTION:** The definitions and conditions described below are made principally for mathematical convenience, and do necessarily not represent market realities for consumers. At best, they approximate the conditions very large investors such as banks, investment houses, governments, and the like, may be able to work under. The conditions for retail investors and brokers are laid out in detail by the Securities and Exchange Commission. See for example <http://www.sec.gov>.

## 2. STOCKS AND THEIR DERIVATIVES

A **derivative** or **contingent claim** is a contract between a **writer** and a **holder**, where the outcome of the contract is to be determined by the future value of some equity. There are many forms such a contract may take. Here are some examples.

**FORWARD CONTRACT:**

The elements are:

- On a specified future date, called the **expiration date** or **strike date**, the holder must pay a specified sum, called the **exercise price** or **strike price** to the writer. The holder may have to pay the writer an initial premium, or the writer may have to pay an initial premium to the holder.
- On the expiration date, the writer must deliver the specified share of the equity to the holder, and the holder must pay the specified price to the writer.

A forward contract is also referred to more simply as a **forward**. A contract holder has the right to trade the forward, as well as to buy new forwards at any time. Suppose  $t = t_0$  is the date the contract is begun, and  $T > t_0$  denotes the strike date. Let  $S_t$  denote the price of the underlying stock at an arbitrary time  $t$ . Let  $x$  denote the strike price, a specified quantity in the contract. Then  $S_T - x$  denotes the profit to the holder at time  $T$ , and its negative is the profit to the writer. (A negative profit is of course a loss.) For the holder, the forward would

seem to be a good bet if the holder expects the stock price to be greater than the strike price by time  $T$ , but if the writer believed this also, the writer would not enter into the contract in the first place.

The price for a forward bought on a forward trading market obviously depends on the value of the underlying equity. At time  $t$  (with  $t_0 \leq t < T$ ), if  $t$  is close to  $T$  and the stock price exceeds the strike price, the price of such a forward will be high, and conversely, it will be low if the stock price is much lower than the strike price. Because a forward is a tradable item, it has some value  $F_t$  for  $t_0 \leq t < T$ , which we discuss below.

#### SELLING SHORT :

Selling a basic equity **short** means that the investor, in effect, writes a contract at time  $t = t_0$  to deliver a share of that basic equity at some future time  $T$ , even if the writer does not currently own that equity. The holder of the short pays the writer some price  $y$  at the contract signing. Other equities may in fact be sold short, as if they were basic equities, so that, for example, a forward contract may be sold short.

#### MECHANICS OF SHORT SELLING:

When you sell a stock short, you follow these procedures:

- You borrow (without a specified replacement date) a specific number of shares, usually from a broker, and sell these shares immediately at price  $y$ .
- If the owner of the borrowed shares decides to sell, you must borrow other shares to replace the first borrowed shares, under the same conditions.
- Normally, you replace the shares on the expiration date, buying them at the current market price, when you hope that prices will be lower.

(The actual process for retail consumers is considerably more complicated and has a number of costs which are not covered by our models.)

#### BUYING LONG:

A short seller is gambling that the price will go down between time  $t_0$  and  $T$ . A **long buyer** is gambling that the price will go up. The mechanics of buying long are simple: buy the stock at time  $t_0$  and hold it until  $T$ .

#### ARBITRAGE:

The term **arbitrage** refers in the first place to opportunities to make money without risk in a market, and one who practices this art is called an **arbitrageur**. The arbitrageur serves the same function in economic markets that the wolf serves in the natural world, watching constantly for opportunities to hold positions that cannot lose under any conditions, and stand a chance of positive gain, so that the expectation of gain is strictly positive, completely without risk. The **principle of no arbitrage opportunities** in the mathematics of finance is a postulate that, because of the way real markets work, true arbitrage opportunities are so rare as to be negligible.

We shall use this principle to analyze the value  $F_t$  at time  $t$  ( $t_0 \leq t \leq T$ ) of a forward with strike price  $x$  expiring at time  $T$ . We begin by imagining building at time  $t_0$  a **portfolio** which **replicates the underlying stock**, whose price at time  $t$  is denoted as usual by  $S_t$ . By this we mean that at time  $t = t_0$ , the portfolio consists of two items—a forward contract and a bank account with value  $xe^{-r(T-t_0)}$ . The term  $r$  is supposed to be the bank interest rate, so the cash in the bank will grow by time  $T$  to  $X$ . At time  $T$ , we have to pay  $X$  to the contract writer and receive a unit of stock worth  $S_T$ , at which point the value of the forward drops in effect to 0. Therefore, the value of the portfolio at time  $T$  is  $S_T$ . We shall argue below based on the principle of **no arbitrage opportunities** that

*forward price at time  $t$  plus cash in bank at time  $t$  must equal the share price at time  $t$ .*

That is,

$$(2.1) \quad F_t + xe^{-r(T-t)} = S_t$$

If at some time  $t < T$  one had  $F_t + xe^{-r(T-t)} > S_t$ , then an arbitrageur would do the following:

- at time  $t$ , sell a forward short (expiring at  $T$ , strike price  $x$ ), immediately gaining  $F_t + xe^{-r(T-t)}$ ;
- at time  $t$ , buy the stock long at price  $S_t$ ;
- at time  $t$ , make an immediate profit in the amount  $F_t + xe^{-r(T-t)} - S_t$ ;
- at expiration, sell the stock at price  $S_T$  and buy the forward contract at cost  $S_T$ , covering the short sale of the forward, at no net cost.

If on the other hand  $F_t + xe^{-r(T-t)} < S_t$ , then an arbitrageur would do the following:

- at time  $t$ , buy the forward (expiring at  $T$ , strike price  $x$ ) at price  $F_t + xe^{-r(T-t)}$ ;
- at time  $t$ , sell the stock short at price  $S_t$ , gaining  $S_t$ ;
- at time  $t$ , make an immediate profit in the amount  $S_t - (F_t + xe^{-r(T-t)})$ ;
- at expiration, the forward will have value  $S_T$ , which the arbitrageur uses to cover the short, at no net cost.

Real markets would not support arbitrageurs making such riskless, guaranteed profits, and it is therefore natural to impose the economic law of “no arbitrage opportunities”, which in this case entails the truth of (2.1).

As a consequence of (2.1), one sees in particular that the value of the replicating portfolio described above is identical to the stock value during its entire duration  $t_0 \leq t \leq T$ , whence its name.

#### EXAMPLE OF FORWARD PRICING:

Let the interest rate be 2% per year (253 trading days). Suppose we wish to find the contract price of a forward on a stock that will expire in 60 trading days, having a current stock price of 52.65, and an exercise price of 54.10.

We measure time  $t$  in trading days, so the interest rate  $r$  per day satisfies  $e^{253r} = 1.02$ , whence

$$r = \log(1.02)/253 \approx .00007827.$$

We'll measure time starting today, so  $t = 0$  and  $T = 60$ . We are also given  $X = 54.1$  and  $S_0 = 52.65$ . From this data, we find  $e^{-r(T-0)} \approx 1.00471$ . The no-arbitrage price equation (2.1)  $F_0 + Xe^{-r(T-0)} = S_0$  then leads to

$$F_0 = S_0 - Xe^{-r(T)} \approx 52.65 - 54.1 * .995315 \approx -1.20.$$

That is, if I wish to enter a forward contract on this stock at the given strike price and expiration date, I should be prepared for a loss of about \$1.20 per share.

#### CALL OPTIONS:

A call option is a contract giving the holder the opportunity (but not the obligation) to buy a share of stock for a specified price at a specified time in the future. More precisely:

- the buyer (holder) pays the seller (writer) a fee for the option, called a premium;
- on the **expiration date**, the holder **may** pay the writer the exercise price, in which case the writer is obliged to deliver to the holder one share of the underlying stock.

If the holder elects to pay the seller the exercise price, one says that the option has been **exercised**.

Clearly, a call option will be exercised if and only if  $S_T \geq X$  and the payoff from exercising the call will be  $S_T - X$ . If  $S_T < X$ , the option will not be exercised, and the payoff from the call will be 0. That is

$$\text{Call payoff} = \max\{S_T - X, 0\} =: (S_T - X)^+.$$

The type of call option described above, where the holder may exercise its use only at the time of expiration, is a **European call option**. There is another type where the holder may exercise the option at any time up to the expiration date. This is termed an **American call option**.

The issue of how to price a call option is much more complicated than for forward options, especially in the case of an American call option. It should be clear though that, because an American call option gives the holder more opportunity to take advantage of a temporary price increase, it should cost the holder more than the corresponding European call option. Likewise, when you compare the workings of a European call option

to the corresponding forward option, the holder of a European option can walk away without suffering the loss if  $S_T < X$ , and so the European call option should cost more. That is:

$$(2.2) \quad \text{forward option price} \leq \text{European call option premium} \leq \text{American call option premium.}$$

#### PUT OPTIONS:

The opposite of a call option is a **put option**. This gives the **holder** an opportunity to sell a share of stock (even if not owned by the holder) for a guaranteed price at a specified future time. The elements are:

- The buyer (holder) pays the seller (writer) a fee, called a **premium**.
- On the expiration date, the holder **may** give the writer a share of stock, or equivalently, the current price of a share of the stock, and in either of these cases, the writer **must** pay the exercise price to the holder.

Just as in the case of a call option, the election to exercise the option is up to the holder, but the share seller is the holder with a put, the writer with a call. Just as with call options, there are European and American variants, the European being the one described above, and the American allowing the holder to exercise the option to sell at any time up to the exercise date, not just on the expiration date. Pricing put options is a problem of similar difficulty to pricing call options. Note that the payoff from a put option is given by

$$\text{Put payoff} = \max\{X - S_T, 0\} = (X - S_T)^+.$$

#### EXAMPLE OF A PUT OPTION:

The following example shows the use of a put option as a kind of insurance that stocks you will need to sell will be priced high enough for your needs. Suppose you hold shares of a stock priced at \$60 that you will need to sell off to finance college expenses. As the price fluctuates, you are concerned that when you need to sell to raise cash, the price might be considerably lower. You decide to buy a series of put options, each costing \$3, expiring at 90 day intervals, with strike price \$55. In this way, every 3 months, depending on whether the stock price is more or less than \$55, you (i) sell stock if its price is higher, not exercising the option, or (ii) exercise the option if the stock price is lower than \$55, receiving \$55 per share. In effect, you are paying an insurance premium of \$3 per share to guarantee a minimum price of \$55 when you need to sell.

### 3. FUTURES CONTRACTS

A **futures contract** between two parties, the **buyer** (or holder) and the **seller** (or writer), is an agreement that on a specified date in the future, an exchange, usually of goods for money, will take place. Nothing changes hands until that future time. Futures may also be written on stocks or other equities.

For example, on January 1, a buyer enters an agreement with a seller to buy one one barrel of oil for a price of \$35 for delivery March 1. The price will be paid when the barrel is delivered.

There is a similarity here to the forwards discussed earlier. In particular, every future contract may be considered to be a forward contract, but as a forward contract may involve an initial cost, not every forward contract may be considered to be a futures contract.

**3.1. Pricing a stock future:** This consideration allows us to set a value on a futures contract on a stock or other equity. If the stock has price  $S_t$  at time  $t$  and the interest rate is  $r$ , we may ask what price  $x$  to put on a future drawn up at time  $t_0$ , expiring at time  $T$ , knowing its current value  $S_{t_0}$ . By the earlier discussion of forwards pricing, the value  $F_t$  of such a forward at time  $t$  is  $F_t = S_t - xe^{-r(T-t)}$ . In order that this vanish at time  $t = t_0$ , so that there is no up-front cost,  $S_{t_0} = xe^{-r(T-t_0)}$ , from which it follows that the no arbitrage price of the future should be  $x = S_{t_0}e^{r(T-t_0)}$ , which is of course just the value that would accrue to a bank account deposit of  $S_{t_0}$  at  $t_0$  and left until time  $T$ .

3.2. **A no-arbitrage argument for stock price growth:** Let  $S_t$  denote the price of a particular stock at time  $t$ ,  $t_0 \leq t \leq T$ . let  $r$  denote the bank interest rate. We are now in a position to argue that

$$(3.1) \quad E \frac{S_T}{S_t} = e^{r(T-t)}; \quad t_0 \leq t \leq T.$$

Suppose first that  $ES_T > S_t e^{r(T-t)}$ , so that it appears at time  $t$  that the stock price is going to grow more than theory says it should. An arbitrageur then does the following:

- at time  $t$ , sell the stock short, expiration date  $T$ , gaining cash  $S_t$  (which will grow to  $S_t e^{r(T-t)}$  by time  $T$ ) and an obligation to replace the stock at time  $T$ ;
- enter into a futures contract to purchase the stock at time  $T$ , at the strike price  $S_t e^{r(T-t)}$ , based on the model above;
- at time  $T$ , use the cash  $S_t e^{r(T-t)}$  to settle the futures contract, gaining in return a share of stock

3.3. **Pricing a commodities future:** Commodities are different from equities in some important respects.

- Commodities cannot be sold short.
- Where there is usually a fixed supply of stocks, large new supplies of commodities are introduced to the market.
- Equities have small storage costs, unlike commodities.

Commodities futures don't satisfy the same simple no-arbitrage pricing rules that equities futures do.

#### 4. BOND MARKETS

A bond is a loan, where the borrower (or seller) promises to repay the loan to the holder at some specific time in the future, plus interest at a specified rate. Bonds may be sold short, just like stocks. A bond has a **face** or **par** value, which is the sum to be received by the holder at maturity. There are two distinct types of bonds—**discount** and **coupon**. Discount bonds are also called **zero-coupon bonds**. The holder is paid only the face value at maturity. All interest is effectively built into the price, which is discounted from the face value. In contrast, coupon bonds provide fixed, periodic interest payments over the lifetime of the bond. When a bond of either type is issued, its price is fixed by the issuer, but from that time until the bond matures, its value is determined by market factors, principally the current risk-free interest rate. The current value of bonds on the secondary market is chiefly influenced by two factors.

- Economic factors that most people would consider positive (employment rates, higher wages) depress existing bond prices because there is a fear that inflation will cause new bond issues to be at a higher interest rate, making older issues less attractive. For the same reason, increases in the prevailing interest rates depress prices for existing bonds.
- Conversely, economic news that most people would consider negative (higher unemployment, stagnating wages) enhance existing bond prices.

4.1. **Rates of Return:** The rate of return on a bond bought new or at market price on the secondary market can be accounted for in several different ways, the last of which seems to be most informative.

- **Coupon rate:** the periodic payment, measured annually, as a percentage of the face value.
- **Current yield:** the annual payout as a percentage of the current market price.
- **Yield-to-maturity:** the rate of return, as an annual percentage, assuming it is bought and held to maturity.

As we've seen before, if the risk-free interest rate is constant and equal to  $r$ , the value  $v$  at time  $t$  of a promise to repay a payoff amount  $p$  at a future time  $T$  is

$$(4.1) \quad v = pe^{-r(T-t)}.$$

For a bond, the yield-to-maturity rate  $R$  is defined to be the rate that, when used to discount all future payoffs to their present value, will yield the present value of the bond. It's easy to solve (4.1) in the case of a zero coupon bond. Suppose such a bond has face value  $p$  and matures at time  $T$ , and that its price at an earlier time  $t$  was  $p_t$ . Substituting in (4.1) gives  $p_t = pe^{-R(T-t)}$ , and solving for  $R$  we find

$$(4.2) \quad R = \frac{-\log(p_t/p)}{T-t}.$$

Coupon bonds are much harder to deal with. We'll ignore them for now.

**4.2. Interest rates:** The  $n$ -year interest rate on a bond investment is defined to be the interest rate on an investment beginning today and lasting  $n$  years. This is the same as the yield-to-maturity of a zero coupon bond issued today and maturing in  $n$  years. The rate may be obtained from (4.2).