

MATH 194, HOMEWORK 2, DUE IN CLASS FRIDAY, APRIL 16

1. Let B_t and C_t be standard Brownian motions, independent of one another in the sense that every random variable B_s is independent of every random variable C_t . Let $D_t := bB_t + cC_t$, where b and c are constants. Show that D_t is a multiple of a standard Brownian motion.
2. Assume that the result in problem 1 has been proven, Compute formulas for $\text{Var}(D_t)$, $\text{Cov}(B_t, D_t)$ and the correlation coefficient $\rho(B_t, D_t)$.
3. Suppose stock prices S_t follows a lognormal model with drift μ and volatility σ . Let r denote the interest rate. Assume that the “no arbitrage principle” applies, so that $\mu + \sigma^2/2 = r$. Fix a time T . Compute (in terms of the standard normal cdf Φ) the probability that $S_T/S_0 > 2e^{T\mu}$. (Recall that S_T/S_0 has median $e^{T\mu}$.) Assuming that T signifies 1 year and the real interest rate is 2.5% per annum, find the largest value this can take, as σ varies.
4. (Attach a printout of the Excel spreadsheet showing your work. It is important to label each calculated row or column so that we know what you have done.) Download CSV tables of at least 100 business days of stock price history (your choice of 10 different stocks, none of which have declared a dividend during the period) from biz.yahoo.com, importing the CSV tables into a spreadsheet. Only the last column will be relevant. From that column, estimate the drift μ and volatility σ of the stock, based on a lognormal model. Then examine the 10 quantities $\mu + \sigma^2/2$. Based on “no arbitrage”, if the lognormal model applies, this quantity should be equal to the real interest rate for all the stocks. Estimate the real interest rate. Examine your data for discrepancies.