MATH 194, HOMEWORK 4, DUE IN CLASS FRIDAY, MAY 7

1. Assume that a finite sample space Ω with probability P has a σ-algebra $\mathcal{F}$ generated by a partition $\mathcal{A}$. Prove that for any random variable X, the random variable $E\{X \mid \mathcal{F}\}$ is the best least squares predictor of X among all $\mathcal{F}$-measurable random variables. (Hint: consider any $\mathcal{F}$-measurable random variable Y, and write expand $E(X-Y)^2$ after subtracting $E\{X \mid \mathcal{F}\}$ from each term inside the parentheses.)

2. Let ξ_1 through ξ_T denote iid random variables, each taking two possible values d, u, with $0 < d < u$, and let $p := P\{\xi_j = u\}$. Use the multiplicative form of the Central Limit Theorem to identify the approximate distribution of $\xi_1\xi_2\ldots\xi_T$, assuming T is large.

3. Exercise 1, section 2.4 of Williams' notes.