10B Exam 2

Feb. 24, 2015

Solutions

1. $$\frac{9}{x^2-7x+10} = \frac{A}{x-5} + \frac{B}{x-2}$$

$x=2$: $B=-3$

$x=5$: $A=3$

$$\int \frac{9}{x^2-7x+10}\,dx = \int \frac{3}{x-5}\,dx - \int \frac{3}{x-2}\,dx$$

$$= \boxed{\ln|x-5| + (-\ln|x-2|)} + C$$

2 a. $$\int 4t\cos(5t)\,dt$$

$u = 4t \qquad v' = \cos(5t)$

$u' = 4 \qquad v = \frac{1}{5}\sin(5t)$

$$= \frac{4t\sin(5t)}{5} - \int \frac{4}{5}\sin(5t)\,dt$$

$$= \boxed{\frac{4t\sin(5t)}{5} + \frac{4}{25}\cos(5t) + C}$$

2(b) $\displaystyle\int \frac{\sec^2(\ln x)}{x}\,dx$

$u = \ln x \qquad \frac{du}{dx} = \frac{1}{x}$

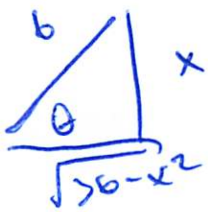
$$= \int \sec^2 u \, du = \tan u + C$$

$$= \boxed{\tan(\ln x) + C}$$

3. $\displaystyle\int \frac{1}{x\sqrt{36-x^2}}\,dx \qquad x = 6\sin\theta \qquad$ "$dx = 6\cos\theta\, d\theta$"

$$= \int \frac{6\cos\theta\, d\theta}{6\sin\theta\sqrt{36-36\sin^2\theta}} = \frac{6}{36}\int \frac{d\theta}{\sin\theta}$$

(Use table given) $\displaystyle = \frac{1}{6}\cdot\frac{1}{2}\ln\left|\frac{\cos\theta - 1}{\cos\theta + 1}\right| + C$



$\cos\theta = \frac{\sqrt{36-x^2}}{6}$

$$= \boxed{\frac{1}{12}\ln\left|\frac{\frac{\sqrt{36-x^2}}{6} - 1}{\frac{\sqrt{36-x^2}}{6} + 1}\right| + C}$$

4. $\int_0^3 \frac{e^x}{e^x-1} = \lim_{r\to 0^+} \int_r^3 \frac{e^x}{e^x-1}$

$u = e^x - 1$

$\frac{du}{dx} = e^x$

$\int \frac{e^x}{e^x-1} = \int \frac{du}{u} = \ln|u| + C = \ln|e^x - 1| + C$

$\int_0^3 \frac{e^x}{e^x-1} = \lim_{r\to 0^+} \ln|e^x - 1| \Big|_r^3$

$= \lim_{r\to 0^+} \ln|e^3 - 1| - \ln|\underbrace{e^r - 1}_{\to 0}|$

$\to \infty$

$\boxed{\text{Integral diverges}}$

5. $x^2 - 4x = 3x$

$x^2 - 7x = 0$ $\qquad$ $x = 0, 7$

$$\text{Area} = \int_0^7 3x - (x^2 - 4x)\,dx = \int_0^7 7x - x^2\,dx$$

$$= \frac{7x^2}{2} - \frac{x^3}{3}\Big|_0^7 = \boxed{\frac{7(49)}{2} - \frac{7^3}{3}}$$