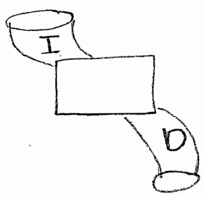
$\boxed{\text{September 24}}$ ~ Chapter 3.1 : Definition of a Function

Let A and B be sets. A function from A to B is a mapping or a rule of correspondences that assigns to each element in A exactly one element in B.

The set A is called the domain. The set of all outputs in B is the range.

something in set A: independent variable
" " set B: dependent variable
↑ depends on what is "plugged in"



D I
$y = x^2$

domain: $\mathbb{R}$ (real #'s)
range: $[0, \infty)$ ← not including
↑ including 0

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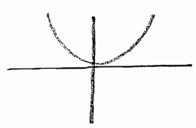
(1) $y = x^2$
yes



(2) $x = y^2$
no if x is dependent

(3) $F = \frac{9}{5}C + 32$
yes

(4) $c^2 = a^2 + b^2$
no

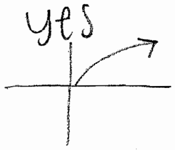
(5)

x	1	5	8	-2
$f(x)$	3	3	2	1

↙ Domain
↑ Range

yes

(6)



yes

(7) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
↑ Dependent yes - multi-variable function

(8) Every # can be written in the form $x = y + \alpha$, where α is in $[0, 1)$
input output
y is an integer
yes

(9)

input Average	output Grade
90-100	A
80-89	B
70-79	C
60-69	D
0-59	F

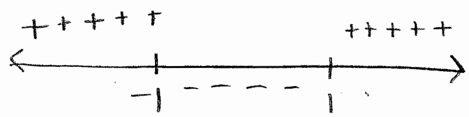
yes

Domains of Functions

$$y = \frac{1}{\sqrt{x^2-1}}$$

Domain: all #'s that won't break the function

Domain is: all values of x such that $x^2 - 1 > 0$



plot what makes denom = 0

soft bc @ -1 fxn = 0

Try (-2): $(-2)^2 - 1 = 3 > 0$

(0): $(0)^2 - 1 = -1 < 0$

(2): $(2)^2 - 1 = 3 > 0$

$\longrightarrow$ $(-\infty, -1) \cup (1, \infty)$

↑ or "and"

Implicit vs. Explicit

If I can write my function as $y = f(x)$ then I say that this representation ($y = f(x)$) is in explicit form.

↑ stuff w/variables, etc in it (NO Y'S)

Explicit: # crunching on the right hand side

Ex of implicit: $y - x^2 = 0$ → $y = x^2$ now Explicit

$y = 3x + 2y$ → $y = -3x$

Breaks: $x = 4$ or $x = -\#$

* Find the domain of

(a) $y = \sqrt{\dfrac{1}{x-4}}$

Domain: all values of x such that $x - 4 \geq 0$ (and $x \neq 4$)

$\rightarrow$ $x - 4 > 0$

Try $(0) = \frac{1}{0-4} = -\frac{1}{4}$

Try $(6) = \frac{1}{2}$

$-$ 4 $+$

$\boxed{\text{Domain: } (4, \infty)}$

(b) $y = \sqrt[3]{\dfrac{1}{x-4}}$

$\boxed{\text{Domain: } (-\infty, 4) \cup (4, \infty)}$ on TEST?

[everything except $x = 4$]

(c) $y = \sqrt{\dfrac{x+3}{x-4}}$

Domain: $\dfrac{x+3}{x-4} \geq 0$ (and $x \neq 4$)

$\boxed{\text{Domain: } (-\infty, -3] \cup (4, \infty)}$

$+$ -3 $-$ 4 $+$

Test $x = -4$: $\dfrac{-4+3}{-4-4} = \dfrac{-1}{-8} = \dfrac{1}{8}$

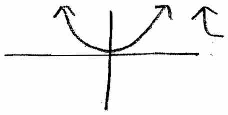
$x = 0$: $\dfrac{0+3}{0-4} = -\dfrac{3}{4}$

$x = 5$: $\dfrac{5+3}{5-4} = \dfrac{8}{1}$

y usually range
x usually domain

Find the Range

$y = \frac{x+2}{x-3}$



Range $(0, \infty)$

$\downarrow$

$y(x-3) = x+2$

$xy - 3y = x+2$

$xy - x = 3y + 2$

$\frac{x(y-1)}{y-1} = \frac{3y+2}{y-1}$

$x = \frac{3y+2}{y-1}$ $\rightarrow$ $\boxed{\text{Range: } (\infty, 1) \cup (1, \infty)}$

$y \neq 1$

Notation

$f(x) = 4 - 3x$

Find

(a) $f(2) = 4 - 3(2)$

$= 4 - 6$

$\boxed{f(2) = -2}$

(b) $f(2x) = 4 - 3(2x)$

$= 4 - 6x$

$\boxed{f(2x) = 4 - 6x}$

(c) $2f(x) = 2(4-3x)$

$\boxed{2f(x) = 8 - 6x}$

(d) $\boxed{f(x^2) = 4 - 3x^2}$

(e) $[f(x)]^2 = (4-3x)^2 \longrightarrow (4-3x)(4-3x)$

$= 16 - 24x + 9x^2$

$\boxed{[f(x)]^2 = 9x^2 - 24x + 16}$

Common Errors

(1) $f(a+b) \neq f(a)+f(b)$

EX: $f(7) \neq f(4)+f(3)$

(2) $f(ab) \neq f(a)f(b)$

(3) $f(1/a) \neq 1/f(a)$

(4) $f\left(\frac{a}{b}\right) \neq \frac{f(a)}{f(b)}$

Try

- $f(x) = 4-3x$

$a=-1$

$b=2$

(1) $f(a+b) \neq f(a)+f(b)$

LHS: $f(1) = 4-3(1) = 1$

RHS: $f(-1)+f(2) = (4-3(-1)) + (4-3(2))$

$= 7 + -2 = 5$

A Difference Quotient

in form $\frac{f(x+h)-f(x)}{h}$ OR $\frac{f(x)-f(a)}{x-a}$

Example:

$f(x) = 4-3x$

Find $\frac{f(x)-f(1)}{x-1} = \frac{(4-3x)-(4-3(1))}{x-1}$

$= \frac{(4-3x)-4+3}{x-1}$

$= \frac{-3x+3}{x-1}$

$= \frac{-3\cancel{(x-1)}}{\cancel{(x-1)}}$

$= \boxed{-3}$