Practice Midterm II of Math 20F *

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Notes: 1)For computational problems, no credit will be given for unsupported answers gotten directly from a calculator. 2)for proof problems, no credit will be given for wrong reasons.

1. Determine whether the following matrix

$$\begin{bmatrix} 1 & -2 & 1 \\ 4 & -7 & 3 \\ -2 & 6 & 4 \end{bmatrix}.$$

   is invertible or not. If so, find the inverse. Show all the computational steps of how you get the answer.

2. Find the determinant of the matrix

$$\begin{bmatrix} 5 & 0 & -1 \\ 1 & -3 & -2 \\ 0 & 5 & 3 \end{bmatrix}.$$

   Show all the computational steps of how you get the answer.

3. Find the dimension and a basis for the null space of matrix

$$\begin{bmatrix} 2 & 4 & -2 & 1 \\ -2 & -5 & 7 & 3 \\ 3 & 7 & -8 & 6 \end{bmatrix}.$$

   Show all the computational steps of how you get the answer.

4. Let $A, P \in \mathbb{R}^{n\times n}$ be invertible matrices. Prove that $\det(PAP^{-1}) = \det(A)$.

5. Let $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ and H be the following set of matrices

$$H = \{X \in \mathbb{R}^{4\times 3} : XB = 0\}.$$

   Determine whether H is a subspace of $\mathbb{R}^{4\times 3}$. Show your reasons.

6. Show that $\{\sin(t)^2, \sin(t)\cdot\cos(t)\}$ is linearly independent in the space $C[0, 2\pi]$. Show your reasons.

*The real exam might be totally different !!!