1. Let $H, K$ bet two subspaces of a vector space $V$. Their sum is

$$H + K = \{ w = u + v : u \in H, v \in K \}.$$ 

Show that $H + K$ is also a subspace of $V$ and

$$\dim(H + K) \leq \dim(H) + \dim(K).$$

2. Let $V$ be a vector space equipped with an inner product $\langle \cdot, \cdot \rangle$. Prove the Cauchy-Schwartz inequality:

$$|\langle x, y \rangle|^2 \leq \langle x, x \rangle \langle y, y \rangle \quad \forall x, y \in V.$$ 

3. Let $x_1, \ldots, x_r$ be a set of orthonormal vectors in $\mathbb{R}^n$. If $Q \in \mathbb{R}^{n \times n}$ is orthogonal, Show that $Qx_1, \ldots, Qx_r$ is also an orthonormal set.

4. Let $T : \mathbb{C}^n \to \mathbb{C}^n$ be a linear mapping and $S \in \mathbb{C}^{n \times n}$ be nonsingular. Suppose the representing matrix of $T$ with respect to a basis $\{u_1, \ldots, u_n\}$ is $A$. Express the representing matrix of $A$ with respect to the new basis $\{Su_1, \ldots, Su_n\}$ in terms of $A$ and $S$.

5. Let $S, T : V \to V$ be two linear mappings over a finitely dimensional vector space $V$. If the product $ST$ is one-to-one, show that both $S$ and $T$ are one-to-one.