1. Suppose $A \in \mathbb{C}^{n \times n}$ is idempoent (i.e., $A^2 = A$) and rank $A = r < n$. Determine the Jordan’s canonical form of $A$.

2. For two matrices $A, B \in \mathbb{C}^{n \times n}$, show that: $A$ is similar to $B$ if and only if $A, B$ have the same Jordan’s canonical form.

3. For every $A \in \mathbb{C}^{n \times n}$, show that there exists an integer $k \leq n$ such that
   \[ \text{rank } A^k = \text{rank } A^{k+1} = \cdots. \]

4. Let $A \in \mathbb{C}^{n \times n}$ and $v \in \mathbb{C}^n$ such that $A^k v = 0$ but $A^{k-1} v \neq 0$. Show that the vectors $v, Av, \ldots, A^{k-1} v$ are linearly independent.

5. For the following matrix
   \[ A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \]
   determine its Jordan’s canonical form (JCF) and find a nonsingular matrix $P$ such that $P^{-1}AP$ gives the JCF.