Math 245A Homework Assignment #2

Instructor: Jiawang Nie

Due Date: October 28, 2016

1. Show that the set

\[ K = \left\{ \frac{X}{1 + \text{Trace}(X)} : X \in S^n, I_n \preceq X \preceq 2I_n \right\} \]

is convex. Is it a spectrahedron? Give reasons.

2. Show that the set

\[ K = \{(X, Y) \in S^n \times S^n : X^2 \preceq \text{Trace}(Y) \cdot Y \} \]

is a spectrahedron.

3. Let \( X \) be the set

\[ X = \left\{ \begin{bmatrix} 1 \\ t \\ t^2 \end{bmatrix} : t \in \mathbb{R} \right\}. \]

Find the convex hull \( \text{conv}(X) \) and show that it is a spectrahedron.

4. Let \( X \) be the set

\[ X = \{(x, t) \in \mathbb{R}^n \times \mathbb{R}_+ : \|x\|_2 = t \}. \]

Show that the convex hull \( \text{conv}(X) \) is the second order cone

\[ \mathcal{L}_n = \{(x, t) \in \mathbb{R}^n \times \mathbb{R}_+ : \|x\|_2 \leq t \}. \]

5. Let \( \mathcal{M} = \{X \in S^n_+ : \text{rank}(X) = 2, \text{Trace}(X) = 1 \} \). Find its convex hull \( \text{conv}(\mathcal{M}) \), affine hull \( \text{aff}(\mathcal{M}) \), and conic hull \( \text{cone}(\mathcal{M}) \).