Math 245A Homework Assignment #3

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Due Date: November 18, 2016

1. Let $A \in \mathbb{R}^{m \times n}$. Show that the inequality

$$Ax < 0$$

has no real solution if and only if the system

$$A^T y = 0, \quad 0 \neq y \in \mathbb{R}^m$$

has a solution.

2. Let $C, D$ be the convex sets

$$C = \{(x_1 + 1)^2 + (x_2 + 1)^2 \leq 1\}, \quad D = \{x_1 > 0, x_2 > 0, x_1x_2 \geq 1\}.$$  

Find the set of all hyperplanes separating $C$ and $D$, that is, find the sufficient and necessary conditions for $(a, b)$ satisfying

$$(a, b) \in \mathbb{R}^2 \times \mathbb{R} : a^T x \leq b, \forall x \in C, \quad a^T y \geq b, \forall y \in D.$$  

3. Find sufficient and necessary conditions for $c_0, c_1, c_2$ satisfying

$$c_0 + c_1x_1 + c_2x_2 \geq 0 \quad \forall (x_1, x_2) \in \begin{cases} 3x_1 + x_2 + 1 \geq 0 \\ x_1 + 3x_2 + 1 \geq 0 \end{cases}.$$  

4. Suppose $C \subseteq \mathbb{R}^n$ is closed, has nonempty interior and has a supporting hyperplane at every point of its boundary. Show that $C$ is convex.

5. Let $f(x) = x^TAx + b^Tx + c$, with $A \in \mathcal{S}^n, b \in \mathbb{R}^n, c \in \mathbb{R}$. If the set \{x : f(x) \leq 0\} \neq \mathbb{R}^n is convex, show that $A \succeq 0$. 

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