1. Denote by $S^2$ the space of real 2-by-2 symmetric matrices. Give a vector space $T \cong \mathbb{R}[x_1, x_2]_2 \otimes S^2$ and prove the isomorphism.

2. Let $V_1, V_2, W$ be three nontrivial vector spaces over a field $\mathbb{F}$. If $V_1 \otimes W \cong V_2 \otimes W$, do we have $V_1 \cong V_2$? If yes, give a proof; if no, explain why.

3. Consider the $3 \times 3$ matrix multiplication map
   $$M_{3,3,3} : \mathbb{C}^{3 \times 3} \times \mathbb{C}^{3 \times 3} \to \mathbb{C}^{3 \times 3}, \quad (A, B) \mapsto AB.$$  
   Write down a tensor in $\mathbb{C}^{3 \times 3} \otimes \mathbb{C}^{3 \times 3} \otimes \mathbb{C}^{3 \times 3}$ that represents $M_{3,3,3}$.

4. For vectors $v_1, \ldots, v_n \in V$, show that $v_1 \wedge v_2 \wedge \cdots \wedge v_n = 0$ if and only if $v_1, \ldots, v_n$ are linearly dependent.

5. If $\{b_1, \ldots, b_n\}$ is a basis of a vector space $V$, show that
   $$\{\text{sym}(b_{i_1} \otimes \cdots \otimes b_{i_m})\}_{1 \leq i_1 \leq \cdots \leq i_m \leq n}$$
   is a basis for $S^m(V)$. 

Math 277A Homework Assignment #1

Instructor: Jiawang Nie

Due Date: October 25, 2017