Math 277A Homework Assignment #2

Due Date: December 3, 2018

1. Let $e_1, e_2, e_3$ be the standard unit vectors in $\mathbb{C}^3$. Determine the rank, border rank of the skew-symmetric tensor $e_1 \wedge e_2 \wedge e_3$.

2. Let $e_1, e_2, e_3$ be the standard unit vectors in $\mathbb{C}^3$. Determine the symmetric rank and symmetric border rank of the tensor $\text{sym}(e_1 \otimes e_2 \otimes e_3)$.

3. Determine the rank, border rank, symmetric rank, and symmetric border rank of the symmetric tensor $A \in S^3(\mathbb{C}^n)$ given such that

$$A_{ijk} = \cos(i + j + k)$$

for all $i, j, k$ in the range.

4. Write down a Waring decomposition for the polynomial $x_1^2 x_2^2$, whose length of sum is the smallest.

5. For a linearly independent set $\{x, y\} \subseteq \mathbb{C}^n$, consider the tensor

$$x \otimes x \otimes y \otimes y + x \otimes y \otimes x \otimes y + x \otimes y \otimes y \otimes x + y \otimes x \otimes x \otimes y + y \otimes x \otimes y \otimes x + y \otimes y \otimes x \otimes x.$$

Determine its symmetric rank and symmetric border rank.