1. For two vectors \( a_1, a_2 \in \mathbb{R}^n \), define the half spaces 
\[
H_1^+ = \{ x \in \mathbb{R}^n : a_1^T x \geq 1 \}, \quad H_2^+ = \{ x \in \mathbb{R}^n : a_2^T x \geq 1 \}.
\]
If \( H_1^+ = H_2^+ \), show that \( a_1 = a_2 \).

2. For \( x = (x_1, \ldots, x_n) \in \mathbb{R}^n \), denote by \( x[i] \) the \( i \)th largest entry of \( x \). Let 
\[
C = \{ x \in \mathbb{R}^n : x[1] - x[n] \leq 1 \}.
\]
Decide whether or not \( C \) is convex. Give reasons to justify your answer.

3. Let \( C \) be the set 
\[
C = \{ (x_1, x_2, x_3, t) \in \mathbb{R}^4 : x_1 \cdot x_2 \cdot x_3 \geq t^3 \}.
\]
Show that \( C \) is convex.

4. Let \( P \) be the polyhedron in \( \mathbb{R}^3 \) given as 
\[
P = \{ (x_1, x_2, x_3) \in \mathbb{R}^3 : -1 \leq x_2 - x_1 \leq x_3 - x_2 \leq x_1 - x_3 \leq 1 \}.
\]
Find all its vertices, facets and edges, if there are any. Give reasons to justify your answer.

5. Consider the convex set 
\[
T = \left\{ x \in \mathbb{R}^3 : \begin{bmatrix} 1 & x_1 + x_2 & x_2 - x_3 \\ x_1 + x_2 & 1 & x_3 - x_1 \\ x_2 - x_3 & x_3 - x_1 & 0 \end{bmatrix} \succeq 0 \right\}.
\]
Does \( T \) have interior? If yes, give defining inequalities for its interior; if no, give defining equations/inequalities for its relative interior. Give reasons to justify your answer.