First Midterm Exam of Math 171B

Due time: **11pm, April 24**

Exam rules:

- All the topics until the class Tuesday (April 21) may appear in the exam. How to prepare the exam? Suggestions: do or review homework assignments, review class notes, read relevant pages in the book, attend your TA’s discussions, and consult in OHs.

- The exam will be in take-home form. The exam questions will be posted in Gradescope, in the afternoon of April 23. The due time is 11pm on April 24, in the local time of California. There is about one and half day to finish. The due time is the same for all students from all time zones in the world.

- Submit your exam solutions through Gradescope, in the same way as you did for homework assignments. All your answers must be written in rational or radical format.

- For computational problems, you need to show how to get the answers. No credit will be given for unsupported answers that is gotten without explanations. For proof problems, no credit will be given for wrong reasons.

- The exam is open for book and class notes. However, anything other than course materials is not allowed. Calculators are allowed for the exam, but you still need to show all computational steps to get your answers. Any kind of computer or electric software (e.g., Maple, Mathematica, Matlab) is not allowed.

- No cheating is allowed!!! You are not allowed to consult any other person (including classmates, friends, family members, etc). All suspected cheating activities may be reported to the academic integrity office for investigation, without notice to the suspected persons.
First Midterm Exam Questions

1. (20 points) Let $\|x\|\star$ be the norm for $x = (x_1, x_2) \in \mathbb{R}^2$ such that

$$\|x\|\star := |x_1 - x_2| + |x_1 + x_2|.$$  

Then we define the matrix norm as

$$\|A\|\star := \max_{x \neq 0} \frac{\|Ax\|\star}{\|x\|\star}.$$ 

Compute the norm $\|A\|\star$ for the following matrix

$$A = \begin{bmatrix} 3 & -4 \\ 5 & -6 \end{bmatrix}.$$ 

2. (20 points) Determine the values for $a$ so that

$$f(x) := 3x_1^2 + 2x_2^2 + ax_1x_2 + 2x_1x_3 + ax_2x_3$$

is a convex function in $x = (x_1, x_2, x_3) \in \mathbb{R}^3$.

3. (20 points) Suppose $f : \mathbb{R}^3 \to \mathbb{R}$ is a function that is twice differentiable. Let $h(x) := f(Ax)$ be the composite function for the matrix

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}.$$ 

If the Hessian $\nabla^2 h(0) = I_3$ (the 3-by-3 identity matrix), what is the Hessian $\nabla^2 f(0)$ of $f$ at the origin? Show all your computational steps.

4. (20 points) Write down the Newton’s method iteration formula for solving the equations

$$2x_1 - x_2 - x_3 + x_1^2 = (2 - x_1)^2, $$

$$-x_1 + 2x_2 - x_3 + x_2^2 = (2 - x_2)^2, $$

$$-x_1 - x_2 + 2x_3 + x_3^2 = (2 - x_3)^2.$$ 

Choose the initial point $(-3, -2, -1)$. Compute the next iteration point. Does the Newton’s sequence converge or not for this equation? If yes, how many iterations does it require to find the correct solution? If no, explain your reasons.

5. (20 points) Suppose a sequence $\{x_k\}$ is generated by the iteration formula

$$x_{k+1} = \frac{2x_kx_{k-1} + 2}{2x_k + x_{k-1}}$$

with the initial points $x_0 = -3, x_1 = -2$. Compute the iteration points $x_2, x_3, x_4$. If the sequence $\{x_k\}$ converges, what is the limit? Does $\{x_k\}$ have superlinear convergence? Justify your answers.