1. (10 points) Consider the feasible set $F$ defined by the following constraints

\[ x_1 + x_2 \geq 4, \quad x_1 + 3x_2 \geq 6, \quad 6x_1 - x_2 \leq 18, \quad 3 \leq x_2 \leq 6, \quad x_1 \geq -1. \]

(a) Express $F$ in the standard form $Ax \geq b$, and find all the corner points of $F$.

(b) For each corner point, find its active set, strictly feasible constraints, active constraints and infeasible constraints.

2. (10 points) Let $F = \{ x : Ax \geq b \}$ be the feasible set of an LP. Suppose $\bar{x} \in F$, $A(\bar{x}) = \{2, 4, 5, 7\}$ and

\[
A_a(\bar{x}) = \begin{bmatrix}
-1 & -8 & 4 & -7 & -3 \\
-4 & 2 & 0 & 4 & -2 \\
-6 & 0 & -3 & 3 & 5 \\
0 & 0 & 5 & 2 & 7
\end{bmatrix}.
\]

Find a direction $p$ along which the residual of the 5th constraint increases, but all the other active residuals remain the same. What is the active set $A(\bar{x} + \alpha p)$ when $\alpha > 0$ is tiny?

3. (10 points) Let $F = \{ x \in \mathbb{R}^n \mid Ax \geq b, Bx = c, Cx \leq d \}$. Suppose $\bar{x} \in F$ and $A\bar{x} = b$, $B\bar{x} = c$, and $C\bar{x} = d$. Characterize the set of feasible directions at $\bar{x}$ for $F$. Justify your answer.

4. (10 points) Let $F$ be the feasible set defined as

\[ F = \{ x \in \mathbb{R}^2 : |x_1| + 2|x_2| \leq 2, |x_1| \leq 1, |x_2| \leq 1 \}. \]

Is the point $\bar{x} = (1, 0)$ feasible for $F$? Is this $\bar{x}$ a vertex? If not, find a vertex from it using the method described in class. Explain your reasons.

5. (10 points) Let $F = \{ x : Ax \geq b \}$ with $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$. Assume $\bar{x} \in F$. Show that, if a nonzero vector $p$ satisfies $Ap \geq 0$, then $F$ is unbounded.