Math 245B Assignment #2

Due Date: March 3, 2017

1. Express the following optimization

\[
\begin{aligned}
    \min & \quad x_1^2 + x_2^2 + x_3^2 \\
    \text{s.t.} & \quad \begin{bmatrix} x_3 & x_1 & x_2 \\ x_1 & 1 & x_3 \end{bmatrix} \succeq 0, \\
                     & \quad x_1 - x_2 + x_3 \geq 3, \\
                     & \quad x_1 - x_2 \geq 2, -x_2 + x_3 \geq 1
\end{aligned}
\]

as an equivalent linear conic optimization problem, either in the primal or dual format. Point out what is the corresponding vector space \( V \), the cone \( K \), and the linear mapping \( A: V \to \mathbb{R}^m \), the objective vector \( c \in V \) and the constant \( b \).

2. Let \( K \) be the monotonicity cone in \( \mathbb{R}^4 \):

\[
K = \{ x \in \mathbb{R}^4 : x_1 \geq x_2 \geq x_3 \geq x_4 \}.
\]

Consider the linear conic optimization problem

\[
\begin{aligned}
    \min & \quad 6x_1 - 3x_2 - 2x_3 - x_4 \\
    \text{s.t.} & \quad x_1 + x_2 + x_3 = 7, \\
                     & \quad x_2 + x_3 + x_4 = 3, \\
                     & \quad x \in K.
\end{aligned}
\]

Formulate the dual optimization problem, and give the optimality conditions. What is an optimizer?

3. Let \( K \) be a cone in \( \mathbb{R}^n \) and \( A \in \mathbb{R}^{m \times n} \). Suppose \( c \in \mathbb{R}^n, \hat{y} \in \mathbb{R}^m \) satisfy \( c - AT\hat{y} \in \text{int}(K^*) \). Show that the set

\[
\{ x \in K : Ax = 0, c^T x \leq 0 \}
\]

consists of the single point of zero.

4. Let \( c, a_1, \ldots, a_m \in \mathbb{R}^n \). If \( c^T x \leq \max_{1 \leq i \leq m} a_i^T x \) for all \( x \geq 0 \), show that there exists \( \lambda := (\lambda_1, \ldots, \lambda_m) \in \mathbb{R}^m \) satisfying

\[
\lambda \geq 0, \quad c^T \lambda = 1, \quad c \leq \sum_{i=1}^m \lambda_i a_i.
\]