Math 171A Homework Assignment #1

Due Date: January 19, 2018

1. (10 points) Find the minimizer of the following LP:

   \[
   \begin{align*}
   \text{minimize} & \quad 7x_1 - 9x_2 \\
   \text{subject to} & \quad 3x_1 + 4x_2 \leq 5, \quad -5x_1 + 3x_2 \leq 4, \quad 4x_1 - 5x_2 \leq 3, \quad -4x_1 - 3x_2 \leq 5.
   \end{align*}
   \]

   Do this by drawing the feasible set and check corner points.

2. (10 points) Consider the following LP:

   \[
   \begin{align*}
   \text{minimize} & \quad 2x_1 + 3x_2 + 4x_3, \\
   \text{subject to} & \quad 3x_1 - 4x_2 - 5x_3 \geq 6, \\
   & \quad x_1 + x_2 + x_3 = 10, \\
   & \quad x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.
   \end{align*}
   \]

   Eliminate the equality constraint by replacing \(x_3\) in terms of \(x_1, x_2\), and convert it into an equivalent LP with only inequality constraints. Then find a minimizer \((x^*_1, x^*_2, x^*_3)\) for this LP.

3. (10 points) Find the maximizer of the following LP:

   \[
   \begin{align*}
   \text{maximize} & \quad 3x_1 + 5x_2 + 7x_3 \\
   \text{subject to} & \quad 2 \leq x_1 + x_2 \leq 3, \\
   & \quad 4 \leq x_2 + x_3 \leq 5, \\
   & \quad x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.
   \end{align*}
   \]

   by using the Matlab function \texttt{linprog}.

4. (10 points) Convert the following optimization problem

   \[
   \begin{align*}
   \text{minimize} & \quad 3|x| + 4|y| \\
   \text{subject to} & \quad 5x + 7y \leq -1
   \end{align*}
   \]

   equivalently into an LP form and find a minimizer.

5. (10 points) Define function \(f(x, y) = \max\{2x + y, x + 2y, x + y\}\). Convert the following optimization problem

   \[
   \begin{align*}
   \text{minimize} & \quad f(x, y) \\
   \text{subject to} & \quad |x| \leq y + 2 \\
   & \quad y \leq 2
   \end{align*}
   \]

   into an LP form and find a minimizer.
6. (10 points) Consider the following LP:

\[
\begin{align*}
\text{minimize} & \quad y - ax \\
\text{subject to} & \quad x + 2y \leq 4, -y + 2x \leq 3, x \geq 0, y \geq 0
\end{align*}
\]

where \(a\) is a constant. For what value of \(a\), the solution(s) of this LP are following points respectively?

\[\begin{align*}
(a) & \quad x = 0, \ y = 0 \\
(b) & \quad x = 2, \ y = 1 \\
(c) & \quad x = \frac{3}{2}, \ y = 0 \\
(e) & \quad \text{all points in } \{(x, y)|2x - y = 3, \frac{3}{2} \leq x \leq 2\} \\
(f) & \quad \text{all points in } \{(x, 0)|0 \leq x \leq \frac{3}{2}\}
\end{align*}\]

7. (10 points) Suppose \(a_1, a_2, a_3\) are linearly independent vectors. Determine whether the vectors \(a_1 + a_2, a_2 + a_3, a_3 + a_1\) are linearly independent or not.