RESEARCH STATEMENT

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Introduction

The focus of my research, broadly speaking, is on the relationship between geometry and topology.

To best describe the nature of the interaction between geometry and topology that strongly motivates my work, let us consider the action of a topological group on a space. Typically, we assume that the group in question encodes the symmetries of an underlying geometry. The space on which it acts is then a moduli space relevant to that geometry. For example, let (M, ω) denote a symplectic manifold, and let G_ω be the group of symplectomorphisms of M. Then notice that G_ω acts on the space $\mathcal{J}_\omega$ of ω-compatible complex structures on M. This is a natural example of a group action motivated by geometry, to which one can apply techniques of equivariant topology in order to obtain information relevant to geometry.

Another aspect of the interaction mentioned above is that one can use goemetry to express the relevance of topological invariants. If for instance one has a topological group G that encodes the symmetry of a certain geometry, then one may consider its action on a universal space X. The equivariant topology of X must then have a geometric interpretation. For example, if we take X to be the universal contractible space with a free G action, then the equivariant cohomology of X is simply the universal characteristic classes for principal G-bundles. Similarly, if one takes X to be the universal space for proper G-actions (see details below), then the equivariant K-theory of X is related to the representation theory of G.

Some of the general topics that I have contributed to in the last few years include:

- (i) Topology of symplectomorphism groups and the space of compatible complex structures on rational ruled surfaces.
- (ii) Topology of Kac-Moody groups and highest weight representation theory.
- (iii) Exotic cohomology theories and applications to the immersion problem for real projective spaces.
- (iv) Classification of seven-manifolds equivalent to sphere bundles over spheres.
- (v) Finite loop spaces and the Browder conjecture.

I remain actively involved in several of the above projects. A complete list of my work may be found in my CV on my website (**www.math.ucsd.edu/∼nkitchlo**). Let me now go into some detail regarding the topics mentioned above.

1. Topology of Symplectomorphism groups and compatible Complex Structures

We begin with the study of the symplectomorphism groups of a family of 4-manifolds. Let M_λ denote the symplectic manifold [1]:

$$(\mathbb{S}^2 \times \mathbb{S}^2,\ \omega_\lambda = \lambda\sigma \times \sigma), \qquad 1 \leq \lambda, \quad \lambda \in \mathbb{R}$$

where σ denotes the standard area form on $\mathbb{S}^2$ with unit area. It is known by work of Lalonde-McDuff [28] that any symplectic form on $\mathbb{S}^2 \times \mathbb{S}^2$ is equivalent to some ω_λ up to scaling.

Let G_λ be the group of symplectomorphism M_λ which act as the identity in cohomology. Gromov [14] used the theory of J-holomorphic curves to study the case $\lambda = 1$. Later, Abreu-McDuff [3] used ideas of Gromov to analyse G_λ for $\lambda > 1$. In particular, they described the rational homotopy type of the classifying space BG_λ for all λ.

McDuff [30] has shown that the homotopy type of BG_λ changes exactly when λ crosses an integer. We may therefore assume that λ is an integer. We shall make this assumption in what follows.

In joint work with M. Abreu and G. Granja [2], I explore the integral homotopy type of BG_λ for all λ. To this end, we study the G_λ-space $\mathcal{J}_\lambda$ of integrable complex structures compatible with the symplectic form ω_λ. The results fit into a beautiful Morse theoretic framework that I will now describe [1].

Donaldson [11] has shown that G_λ acts via symplectomorphisms on the infinite dimensional symplectic manifold of compatible almost-complex structures on M_λ. Recall that and almost-complex structure J on M_λ is compatible if the bilinear form $\omega_\lambda(Jx, y)$ is a J-invariant metric on M_λ. Donaldson has also constructed a moment map for this action. Recall that the Lie algebra of G_λ can be identified with the space of smooth functions on M_λ which integrate to zero on M_λ. Let $C_0^\infty(M_\lambda)$ denote this space. Note that $C_0^\infty(M_\lambda)$ has an invariant inner product given by

$$\langle f, g\rangle = \int f g\, \omega_\lambda^2$$

Donaldson constructs the moment map μ for the action of G_λ on $\mathcal{J}_\lambda$:

$$\mu : \mathcal{J}_\lambda \longrightarrow C_0^\infty(M_\lambda), \qquad \mu(J) = S(J)$$

where $S(J)$ is the scalar curvature of the metric corresponding to J. Suggested by results of F. Kirwan [20] in the finite dimensional setting, one expects the norm-square of the moment map to be a G_λ-invariant Morse-Bott function on $\mathcal{J}_\lambda$. Therefore, one is led to analyse the map

$$|\mu|^2 : \mathcal{J}_\lambda \longrightarrow \mathbb{R}, \qquad |\mu|^2(J) = \int S(J)^2\, \omega_\lambda^2$$

We have analysed the ciritical values of this functional and shown that there are λ such values $c_0 < \ldots < c_{\lambda-1}$. Moreover, the critical level-set corresponding to c_n

[1] All results have analogs for the case of the nontrivial $\mathbb{S}^2$ bundle over $\mathbb{S}^2$

is a single G_λ-orbit G_λ/K_n, where K_n is a compact Lie group. One may also give meaning to the unstable subspace U_n at the critical sets with value c_n in terms of the complex structures on M_λ. By results of Abreu, and McDuff [3, 29] we know that the contractible space of compatible almost-complex structures on M_λ admits a similar stratification. On comparing these stratified spaces, we proved the following result:

Theorem [2]**.** *The space $\mathcal{J}_\lambda$ of compatible complex structures on M_λ is contractible. In addition, there are isomorphisms:*

$$H^*(BG_\lambda, \mathbb{Z}) = H^*(BK_0, \mathbb{Z}) \bigoplus_{i=1}^{\lambda-1} \Sigma^{4i-2} H^*(BK_i, \mathbb{Z})$$

$$H^*(BG_\lambda, \mathbb{Q}) = \frac{\mathbb{Q}[x, y, z]}{\langle z \prod_{i=1}^{\lambda-1} (z^2 + i^4 x - i^2 y) \rangle}.$$

The space of compatible complex structures on symplectic manifolds is an elusive object, and it is a hard geometric problem to describe its topology. It is therefore quite encouraging that topological methods may be used along with geometric insight to completely solve this problem in this instance.

One may ask the question as to when the space of compatible complex structures is contractible for a symplectic manifold. This is a very interesting question which I am actively pursuing.

2. Topology of Kac-Moody groups and highest weight representations

In recent years a class of topological groups known as loop groups (see [31]) has received considerable attention. Since their introduction, loop groups have played an increasingly important role in mathematics and mathematical physics. It is becoming clear that the representation theory of loop groups plays a crucial role in the geometric description of the elusive equivariant Elliptic cohomology (in much the same way as the representation theory of Lie groups plays a crucial role in the description of equivariant K-theory). The class of Lie groups and loop groups has been further extended by V. Kac and D. Peterson (see [18, 19]) to include certain infinite dimensional topological groups known as Kac-Moody groups. These groups form a natural extension of the class of compact Lie groups, and share many of their properties. In particular, the theory of highest weight representations has an analog in the world of Kac-Moody groups, known as highest weight integrable representations. For a loop group, these representations are none other than the positive energy representations.

Kac-Moody groups are infinite dimensional in nature and thus fall beyond the scope of the usual techniques of geometry. The infinite analytic nature of Kac-Moody groups makes their study a highly nontrivial task. The project of understanding the topology of their classifying spaces using algebraic invariants, such as homology and cohomology, was begun in my thesis [21, 22]. In joint work [8], I have succeeded in obtaining new results about the structure of these groups.

The representation theory of Kac-Moody groups has been extensively studied through the theory of highest weight representations of their lie algebras. It is natural to ask if these representations can be encoded topologically through a variant of equivariant K-theory.

Given a compact Lie group $\mathcal{G}$ and a compact $\mathcal{G}$-space X, the equivariant K-theory of X, $K_{\mathcal{G}}(X)$ has a well-known description as the Grothendieck group of equivariant vector bundles on X. If one tries to relax the condition of $\mathcal{G}$ being compact, one immediately runs into technical problems in the definition of equivariant K-theory. One way around this problem is to control the action of the group $\mathcal{G}$ on the space X. By a proper action of a topological group $\mathcal{G}$ on a space X, we shall mean that X has the structure of a $\mathcal{G}$-CW complex with compact isotropy subgroups. For such a $\mathcal{G}$-space X, the equivariant K-theory may be defined. There exists a universal space with a proper $\mathcal{G}$ action, known as the classifying space for proper $\mathcal{G}$-actions, which is unique up to $\mathcal{G}$-equivariant homotopy. If $\mathcal{G}$ is a compact Lie group, then $\underline{E\mathcal{G}}$ is simply equivalent to a point, and so the equivariant K-theory of $\mathcal{G}$ is isomorphic to the representation ring of $\mathcal{G}$. For a general non-compact group $\mathcal{G}$, the geometric meaning of $K_{\mathcal{G}}(\underline{E\mathcal{G}})$ remains unclear.

An important result of Freed-Hopkins-Teleman [12] should be mentioned in this context: Let G denote a compact Lie group, and let $\mathbb{L}G$ denote the universal central extension of the group LG of free loops on G. In this case, the space $\underline{E\mathbb{L}G}$ has a rather easy description as the affiine space of principal G-connections on the trivial G-bundle $G \times S^1$. In [12], the authors show that the equivariant K-theory of the loop group $\mathbb{L}G$ is the Grothendieck group of positive energy representations of $\mathbb{L}G$ (known as the Verlinde Algebra). The positive energy representations form an important class of (infinite dimensional) representations of $\mathbb{L}G$ [31] that have relevance to mathematical physics.

In [23], I have given a simple description of $\underline{E\mathcal{G}}$ for an arbitrary Kac-Moody group $\mathcal{G}$, that makes it possible to calculate the equivariant K-theory with the help of a spectral sequence. I have shown that:

Theorem [23]**.** *If $\mathcal{G}$ is a compact Hyperbolic Kac-Moody group, then the Grothendieck group of highest weight representations of $\mathcal{G}$ is isomorphic to $K_{\mathcal{G}}(\underline{E\mathcal{G}})$.*

A compact Hyperbolic Kac-Moody group is one whose proper parabolic subgroups are of classical type.

Since $\underline{E\mathcal{G}}$ is a universal space for proper actions, the above result describes highest weight representations as K-theoretic Characteristic Classes for all spaces with a proper action of a Kac-Moody group. This becomes relevant in the example of the Hyperbolic Kac-Moody group E_{10} (which is not compact Hyperbolic). The real form of this group has been suggested as the group of symmetries in syper-gravity [7, 10]. In [23], I have completely described the K-theory of E_{10}.

3. Universal Moduli space of Flat connections

Ever since the recent breakthrough by I. Madsen and M. Weiss on the Mumford conjecture, there has been renewed interest in the moduli space of Riemann surfaces.

Let G be a fixed, connected, semisimple, compact Lie group G. Our goal is to study the moduli spaces of flat G-connections on principal bundles over Riemann surfaces. By allowing the complex structures of the surfaces to vary, we are able to prove a stability theorem in homology, and to study a cobordism category built out of such moduli spaces.

The study of moduli spaces of flat connections, and its connection with holomorphic bundles on Riemann surfaces goes back to the seminal work of Atiyah-Bott [4]. Given a Riemann surface Σ without boundary, and a principal G-bundle E, Atiyah and Bott studied the space of holomorphic structures on the complexification E^c. This space admits a canonical (complex-gauge) equivariant stratification that was first described in the work of Harder-Narasimhan [17]. The open-dense stratum for this stratification is also known as the space of semistable complex structures on E^c. The codimension of the remaining strata grows linearly in the genus, and hence the semistable stratum approximates the whole space increasingly with genus of the curve.

The space of holomorphic structures may be identified with the space of principal connections on E. Atiyah and Bott also study the Yang-Mills functional on this space of all connections. They show that the Yang-Mills functional behaves like a perfect, (gauge) equivariant Morse-Bott function, with critical subspace given by the Yang-Mills connections. The analytical aspects of the Yang-Mills flow were not studied by Atiyah and Bott in [4]. However, the authors do motivate the reason why the Harder-Narsimhan stratification represents the descending strata for the critical level sets of the Yang-Mills functional. In particular, this suggests that the open stratum (identified with semistable complex structures on E^c) must equivariantly deform onto the space of minimal Yang-Mills connections on E. This minima can be described in terms of cental connections (see the final section). In the setting where G is semisimple, these Yang-Mills minima are simply the flat connections.

The Morse theoretic program suggested above was completed by G. Daskalopoulos and J. Råde. The authors succeeded in proving the long time convergence of the Yang-Mills flow, thereby rigorously establishing the correspondance between the Yang-Mills moduli spaces, and the moduli spaces of semistable complex structures.

In [9], we consider the moduli space $\mathcal{M}_g^G$ of flat connections on bundles, parametrized over the moduli space of Riemann surfaces of a fixed genus equal to g. Our main theorem about this moduli space yields the homology $H_q(\mathcal{M}_g^G)$ for $2q + 4 \leq g$. For simplicity, we will work over $\mathbb{Q}$. Let us first introduce some notation:

For a graded vector space V over the rationals, let V_+ be positive part of V, i.e.

$$V_+ = \bigoplus_{n=1}^{\infty} V_n.$$

Let $A(V_+)$ be the free graded-commutative $\mathbb{Q}$-algebra generated by V_+. Given a basis of V_+, $A(V_+)$ is the polynomial algebra generated by the even dimensional basis elements, tensor the exterior algebra generated by the odd dimensional basis elements. Let $\mathcal{K}$ be the graded vector space $H^{*-2}(\mathbb{CP}^\infty;\mathbb{Q})$. It is generated by one basis element, κ_i, of dimension $2i$ for each $i \geq -1$. Explicitly, κ_{-1} is a generator, and $\kappa_i = c_1^{i+1}\kappa_{-1}$, for $c_1 = c_1(L) \in H^2(\mathbb{CP}^\infty)$.

Theorem. [9] *There is a homomorphism of algebras,*

$$\Theta : A((\mathcal{K} \otimes H^*(BG;\mathbb{Q}))_+) \longrightarrow H^*(\mathcal{M}^G_{g,\gamma};\mathbb{Q})$$

which is an isomorphism in dimensions less than or equal to $(g-4)/2$.

Our second main result concerns the cobordism category of surfaces with flat connections. We call this category $\mathcal{C}^F_G$ whose objects are closed, oriented one-manifolds S equipped with connections on the trivial principal bundle $S \times G$, and whose morphisms are surface cobordisms Σ between the one-manifold boundary components, equipped with flat G bundles $E \to \Sigma$ that restrict on the boundaries in the obvious way. (See [9] for a precise description).

Our result explicitely identifies the homotopy type of the geometric realization of this category with an infinite loop space. The cohomology of this loop space is computable and as before, it plays the part of characteristic classes for any linear functor defined on the category $\mathcal{C}^F_G$. There are many natural examples of such functors appearing in Mathematical Physics under the general framework of *Two dimensional Gauged Field Theories.*

4. Exotic cohomology theories and applications to the immersion problem for real projective spaces

One way to approach a question in topology is to express it algebraically using a suitable cohomology theory. Consider for example the vectorfield problem, that asks for the maximum number of independent vectorfields on a sphere. The original solution to this problem was given by Frank Adams using secondary operations in singular cohomology. As a consequence of this theorem, one obtains the famous result that the only parallelizable spheres are $\mathbb{S}^1, \mathbb{S}^3$ and $\mathbb{S}^7$. A much easier proof of this corollary can be given using only primary operations in complex K-theory. This is a typical instance of the fact that certain cohomology theories are better suited than others at addressing particular questions in topology.

The topological question that interests us in this section pertains to the minimum dimension of the Euclidean space in which $\mathbb{RP}^n$ may be immersed. This question has a long history starting in the 1950's with the work of John Milnor using characteristic classes. Some of most recent results on the subject make use of the cohomology theory tmf [5]. A table of known immersions, along with a comprehensive list of references has been compiled by D. Davis at: http://www.lehigh.edu/∼dmd1/immtable. Using the tools developed jointly with S. Wilson in [25, 26], we can now give new results based on the brand-new cohomology theory $ER(2)$.

The cohomology theory mentioned above is part of a family of cohomology theories called the periodic real Johnson-Wilson theories $ER(n)$. The cohomology theory $ER(n)$ is the fixed point theory of an involution acting on a generalization of complex K-thoery, known as Johnson-Wilson theory $E(n)$. The theories $ER(n)$ were constructed and developed by Hu-Kriz (see for example, [16]), in particular, they were shown to be $2^{n+2}(2^n-1)$-periodic. We further studied them in [24, 25, 26].

The traditional approach for finding a lower bound on the immersion dimension uses the technique of axial maps: Assuming that $\mathbb{RP}^n$ immerses in $\mathbb{R}^k$, it can then be shown using standard techniques in topology that one can construct an (axial) map:

$$\alpha : \mathbb{RP}^{2^N-2-k} \times \mathbb{RP}^n \longrightarrow \mathbb{RP}^{2^N-n-2},$$

for some sufficiently large value of N. This map has the property of restricting to an isomorphism on the fundamental group of each of the factors in the source. The technique of axial maps involves showing that such an immersion cannot exist by applying a suitable cohomology theory to the map α, and deriving a contradiction.

It is clear that to apply this technique using the cohomology theory $ER(2)$, we will need to calculate the rings $ER(2)^*\mathbb{RP}^m$ and $ER(2)^*(\mathbb{RP}^m \times \mathbb{RP}^n)$ for all integers m, n. These rings are rather complicated in general. However, we may invoke a strong computational tool given by the Bockstein spectral sequence [26]. The Bockstein spectral sequence is constructed from the exact couple obtained from the fibration [25]:

$$\Sigma^{17} ER(2) \xrightarrow{\cup x} ER(2) \longrightarrow E(2)$$

where x is a 2-torsion class in $ER(2)^{-17}(\mathbb{S}^0)$ such that $x^7 = 0$.

This spectral sequence is very helpful in calculating the $ER(2)$ cohomology of products of real projective spaces. In [26] we have used this tool to calculate the $ER(2)$-cohomology of $\mathbb{RP}^n$ for all $n \leq \infty$, and derived various new infinite families of non-imersion results for real projective spaces.

The above results suggest that the real Johnson-Wilson theories $ER(n)$ are useful cohomology theories that are amenable to computation. We have demonstrated their use by applying it to the immersion problem of real projective spaces. However, we belive that they are powerful enough to go far beyond this problem and should find their place in the toolbox of the working topologist. Jointly with a Ph.D student of mine, and S. Wilson, I am presently exploring these new theories.

5. Classification of Manifolds

In joint work with K. Shankar, I have constructed topological invariants using techniques of homotopy theory and surgery theory to identify all 7-manifolds up to PL type that are equivalent to $\mathbb{S}^3$ bundles over $\mathbb{S}^4$. This result may be generalized to higher dimensional manifolds, and is a project to undertake in the future. As a result of our work on 7-manifolds, and subsequent joint work with S. Goette [13], we give a complete solution to an important question in differential geometry pertaining to the Berger space. The Berger space may be described as a homogeneous space for an exotic embedding of Sp(1) within Sp(2). It is known that this space admits a metric of positive sectional curvature. In [15], the question was asked whether the Berger space was diffeomorphic to an $\mathbb{S}^3$ bundle over $\mathbb{S}^4$. In [13] we gave an affirmative answer to this question hence providing the first known nontrivial example of a metric with positive sectional curvature on an $\mathbb{S}^3$ bundle over $\mathbb{S}^4$.

6. Finite Loop spaces and the Browder conjecture

More recently, in joint work with T. Bauer, D. Notbohm and E. Pedersen [6], I have applied the theory of p-compact groups to settle an old conjecture about quasi-finite loop spaces. A quasi-finite loop space is a space L which has finitely generated homology and which is homotopy equivalent to a loop space. One of the motivating questions for surgery theory was whether every finite H-space is homotopy equivalent to a Lie group. This question was answered in the negative by Hilton and Roitberg's discovery of some counterexamples. This prompted a conjecture of W. Browder suggesting that any finite H-space has the homotopy type of a manifold. In [6] we settle this conjecture for quasi-finite loop spaces by showing that they have the homotopy type of smooth parallelizable manifolds.

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