Note Title 1/23/2009

p odd prime, $p \nmid a$ and χ not principal char. mod p

$$g_a(\chi) = \sum_{j=0}^{p-1} \chi(j) e^{2\pi i a j/p}$$

$$g_a(\chi) = g_1(\chi)\,\chi(a') \quad \text{where}$$

$$a'a \equiv 1 \bmod p.$$

$$|g_a(\chi)|^2 = p.$$

<u>Cor.</u> If χ is real non principal then $g_a(\chi)^2 = (-1)^{\frac{p-1}{2}} p$.

Know $\chi(j) = \left(\frac{j}{p}\right)$.

$$g_a(\chi) = \left(\frac{a}{p}\right) g_1(\chi)$$

$$g_a(\chi)^2 = \left(\frac{a}{p}\right)^2 g_1(\chi)^2 = g_1(\chi)^2$$

$$g_1(\chi)^2 = \sum_{j=0}^{p-1} \chi(j)\, e^{2\pi i j/p} \cdot \sum_{k=0}^{p-1} \chi(k)\, e^{2\pi i (-k)/p}$$

$$k = (-1)k = -k \qquad \chi(-k) = \chi(-1)\chi(k)$$

$$= (-1)^{\frac{p-1}{2}} \chi(k) \qquad \overline{\chi(k)} = \chi(k)$$

$$g_1(x)^2 = (-1)^{\frac{p-1}{2}} |g_1(x)|^2 = (-1)^{\frac{p-1}{2}} p.$$

Quadratic reciprocity:

let $\zeta = e^{2\pi i/q}$ $\quad$ q odd prime

$$\mathbb{Z}[\zeta] = \{a_0 + a_1\zeta + \dots + a_r\zeta^r \mid a_i \in \mathbb{Z}\} \subset \mathbb{C}$$

Closed under addition and multiplication in $\mathbb{C}$.

Cyclotomic integers

Theorem. If $z \in \mathbb{Q}$ and $z \in \mathbb{Z}[\zeta]$ then $z \in \mathbb{Z}$.

Lemma: $a^p + b^p \equiv (a+b)^p \mod p$ for $a, b \in \mathbb{Z}$.

$$\frac{1}{x+y} = \frac{1}{x} + \frac{1}{y}$$

Quiz ~~T~~ Wed