Oct 30, 2013

Read Ch 6.7 in Hannett's

$\mathcal{H}_1 \otimes \cdots \otimes \mathcal{H}_n$ Hilbert space

$\exists \; \psi$ a state not entangled $\iff \psi = \psi_1 \otimes \cdots \otimes \psi_n$

In projective space, the set $\{ \psi_1 \otimes \cdots \otimes \psi_n \}$ is closed

Corresponds to $\mathbb{F}^d_{1 \times 1} \times \cdots \times \mathbb{F}^d_{m \times 1}$

Not (Not entangled) = entangled

Meyer: $\Delta$ a function (real poly. of deg. 4)

Property: $\Delta(\psi) \geq 0 \iff \psi$ is entangled

$\Delta(\psi) > 0$

$\mathcal{H}_i$ has basis $\{ |0\rangle, \ldots, |d_i - 1\rangle \}$, $d_i = d^{m_i} d_i$

\[
\psi = \sum_{j=0}^{d_i - 1} |j\rangle \otimes \psi_j
\]

$\Delta(\psi) = \sum_{1 \leq i < j \leq d_i - 1} ||\psi_i \wedge \psi_j||^2$

$\mathcal{H}_2 \otimes \cdots \otimes \mathcal{H}_n$ Hilbert space

Now \[
\psi = \sum_{j=0}^{d_2 - 1} |j\rangle \otimes \psi_{2j}
\]

$\Delta_2(\psi) = \sum_{i < j} ||\psi_{2i} \wedge \psi_{2j}||^2$
Then \( \Sigma_{j=0}^{d_k-1} l_j \otimes v_{kj} \)

\[ \Omega(v) = \sum_{k=1}^{n} \sum_{0 \leq j < d_k} \| v_{ki} \wedge v_{kj} \|^2 \]

Meyer-Wallach measurement of entanglement

Suppose \( \Omega(v) = 0 \), \( \Omega_k(v) = 0 \) for all \( k \)

\( v_{ki}, v_{kj} \) are linearly dependent

\( \exists k \) s.t. \( v_{ki} = y_{ki} z_k, \quad v_{kj} = y_{kj} z_k \)

\[ \sum_{j} l_j \otimes y_j z_k = \sum_{w_k} (y_j|j\rangle \otimes z_k) \]

\[ [v] = [w_1 \otimes \ldots \otimes w_n] \]

Consider Grover's algorithm

uniform \( |j\rangle \)

\( v_0 \rightarrow v_1 \rightarrow \ldots \rightarrow v_k \rightarrow \ldots \)

\[ \Omega(v) = n \| v \|^2 - \sum \text{tr}(v_{ji} v_{ji}) \]

interchange 1 \( \leftrightarrow \) 2

\[ \| v \| = \sum_{j=1}^{n} l_j |v_j| \]
True by a formula of Langrange:

\[ |<v|w>|^2 = |v||w|^2 - |v\wedge w|^2 \]
\[ = <v|v> + <w|w> - |v\wedge w|^2 \]
\[ |v\wedge w|^2 = <v|v><w|w> - |<v|w>|^2 \]

i.e. \[ \|v, \otimes \ldots \otimes v_k\|^2 = \prod \|v_i\|^2 \ldots \]

maximizes on states

\[
\begin{array}{c}
C^2 \otimes C^2 \otimes C^2 \otimes C^2 \\
\uparrow \otimes \downarrow \otimes \downarrow \otimes \downarrow
\end{array}
\]

Why studying entanglement is important?

2. It yields quantum error correction.

\[
\begin{array}{c}
\frac{1}{2}\sqrt{2}(000+111) \otimes (\ldots) \otimes \ldots \\
\text{GHT} \quad \text{GHT} \quad \text{GHT}
\end{array}
\]

Shor's error correcting code.

Look at ways of measuring entanglement.

\[ H_1 \otimes \ldots \otimes H_n \quad \text{dim} H_i < \infty \]

\[ G_1, G_n \]

\[ G = \text{SL}(d_1, C) \times \ldots \times \text{SL}(d_n, C) \]

\[ f : H_1 \otimes \ldots \otimes H_n \rightarrow C \]

st. \[ f((g_1 \otimes \ldots \otimes g_n)(v)) = f(v) \]

called Kempf-Ness theorem in geometric invariant theory.
Invariant called \textbf{SLOCC}

Look at $H_1 \otimes H_2$, want SLOCC invariant

\[ C^n \otimes C^n \text{ ask: what is the measure entanglement?} \]

if $m \neq n$, none

\[ C \otimes C^n = C^n \]

two orbit: 0, $C^n$

\[ \therefore \text{no invariant.} \]

\[ \therefore \text{only invariant: } n=m \]

\[ \text{State } \sum_{i,j=1}^{n} d_{ij} |i\rangle \otimes |j\rangle \]

\[ \det(g_j A g_i^T) = \det(A) \text{ degree } n \]

generator of all of the invariant

\[ \text{every inv. has degree divisible by } n. \]

\[ H_1 \otimes \ldots \otimes H_n \]
\[ d_1 \ldots d_n \]

\[ \text{SL}(n, \mathbb{C}) \text{ contains } E_1, E_n = I \]

degrees of invariant must be multiples of $\text{lcm}(d_1, \ldots, d_n)$