Oct 30, 2013

Read Ch. 6.7 in Hannabuss

$\mathcal{H}_1 \otimes \cdots \otimes \mathcal{H}_n$ Hilbert space

$[v]$ a state not entangled $\Leftrightarrow v = v_1 \otimes \cdots \otimes v_n$

In projective space, the set $\{v_1 \otimes \cdots \otimes v_n\}$ is closed
corresponds to $\mathbb{P}^{d_1-1} \times \cdots \times \mathbb{P}^{d_n-1}$

Not (Not entangled) = entangled

Meyer: Q a function (real poly. of deg 4)

property: $Q(v) \neq 0 \Leftrightarrow v$ is entangled
$Q(v) \geq 0$
$\mathcal{H}_i$ has basis $|0\rangle, \ldots, |d_i - 1\rangle$, $d_i = \dim \mathcal{H}_i$

$$v = \sum_{j=0}^{d_j-1} |j\rangle \otimes v_j$$

$$Q_1(v) = \sum_{1 \leq i < j \leq d_1 - 1} \|v_i \wedge v_j\|^2$$

$\mathcal{H}_2 \otimes \cdots \otimes \mathcal{H}_n$ Hilbert space

Now $v = \sum_{j=0}^{d_2-1} |j\rangle \otimes v_{2j}$

$$Q_2(v) = \sum_{i<j} \|v_{2i} \wedge v_{2j}\|^2$$

Then $\quad v = \sum_{j=0}^{d_k-1} |j\rangle \otimes v_{kj}$

$$Q(v) = \sum_{k=1}^{n} \sum_{0 \le i < j \le d_k - 1} \| v_{ki} \wedge v_{kj} \|^2$$

Meyer-Wallach measurement of entanglement

Suppose $Q(v)=0$, $Q_k(v)=0$ for all k

v_{ki}, v_{kj} are linearly dependent

$\exists z_k$ s.t. $v_{ki} = y_{ki} z_k$, $\quad v_{kj} = y_{kj} z_k$

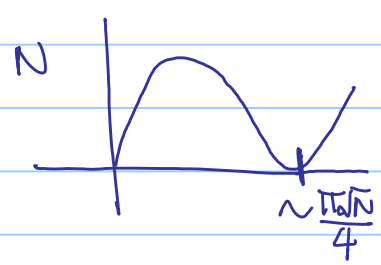
$$\sum |j\rangle \otimes y_{kj} z_k = \sum \underbrace{\left((y_{kj} | j\rangle \right)}_{w_k} \otimes z_k$$

$$[v] = [w_1 \otimes \cdots \otimes w_n]$$

Consider Grover's algorithm

uniform $|j_0\rangle$

$v_0 \to v_1 \to \cdots \to v_k \to \cdots$



$$Q(v) = n \|v\|^4 - \sum \operatorname{tr}(\operatorname{tr}_{\hat{j}}(v)^2)$$

interchange 1 & j

$$\|v\| = \sum_{j=1}^{n} L_j(v)$$

True by a formula of Langrange:

$$|\langle v|w\rangle|^2 = \|v\|^2 + \|w\|^2 - \|v\wedge w\|^2$$
$$= \langle v|v\rangle + \langle w|w\rangle - \|v\wedge w\|^2$$

$$\|v\wedge w\|^2 = \langle v|v\rangle\langle w|w\rangle - |\langle v|w\rangle|^2$$

i.e. $\|v_1\otimes\cdots\otimes v_k\|^2 = \prod \|v_i\|^2 \ldots$

maximizes on states

$$\underbrace{\mathbb{C}^2\otimes\mathbb{C}^2}_{v\otimes w}\otimes\mathbb{C}^2\otimes\mathbb{C}^2$$

Why studying entanglement is important?

① Clearly non-classical.

② It yields quantum error correction.

$$\frac{1}{2^{3/2}}\underbrace{(000+111)}_{GHZ}\otimes\underbrace{(\cdots)}_{GHZ}\otimes\underbrace{\cdots}_{GHZ}$$

Shur's error correcting code.

<u>Look at</u> ways of measuring entanglement.

$$\underset{G_1}{\mathcal{H}_1}\otimes\cdots\otimes\underset{G_n}{\mathcal{H}_n} \qquad \dim\mathcal{H}_i<\infty$$

$$G = SL(d_1,\mathbb{C})\times\cdots\times SL(d_n,\mathbb{C})$$

$$f:\mathcal{H}_1\otimes\cdots\otimes\mathcal{H}_n\to\mathbb{C}$$

s.t. $f((g_1\otimes\cdots\otimes g_n)(v)) = f(v)$

called Kempf-Ness thm in Geometric invariant theory

invariant called <u>SLOCC</u>

look at $\mathcal{H}_1 \otimes \mathcal{H}_2$, want SLOCC invariant

want to study LOCC

$\mathbb{C}^m \otimes \mathbb{C}^n$ ask: what is the measure entanglement?

if $m \lneq n$, none

$\mathbb{C} \otimes \mathbb{C}^n = \mathbb{C}^n$

two orbit : 0, $\mathbb{C}^n$

$\therefore$ no invariant.

$\therefore$ only invariants : $n=m$

state $\sum_{i,j=1}^{n} a_{ij} |i\rangle \otimes |j\rangle$

$\det(g_1 A g_2^T) = \det(A)$ degree n

generator of all of the invariant

every inv. has degree divisible by n.

$\mathcal{H}_1 \otimes \cdots \otimes \mathcal{H}_n$

$d_1 \quad \cdots \quad d_n$

$SL(n, \mathbb{C})$ contains ξI, $\xi^n = I$

degrees of invariant must be multiples of $\operatorname{lcm}(d_1, \ldots, d_n)$