```
In[1]:= << algeom.m

In[9]:= L[n_] := Block[{A = {}, i, j},
          For[i = 1, i < n, i++, For[j = i + 1, j ≤ n, j++,
            A = Append[A, x[i] x[n + j] - x[j] x[n + i]]]];
          A]

In[10]:= L[3]

Out[10]= {-x[2] x[4] + x[1] x[5], -x[3] x[4] + x[1] x[6], -x[3] x[5] + x[2] x[6]}

In[4]:= L[4]

Out[4]= {-x[2] y[1] + x[1] y[2], -x[3] y[1] + x[1] y[3], -x[4] y[1] + x[1] y[4],
         -x[3] y[2] + x[2] y[3], -x[4] y[2] + x[2] y[4], -x[4] y[3] + x[3] y[4]}

In[11]:= G = GroebnerBasis[L[3], Array[x, 6]]

Out[11]= {-x[3] x[5] + x[2] x[6], -x[3] x[4] + x[1] x[6], -x[2] x[4] + x[1] x[5]}

In[12]:= M = MkLeads[G, Array[x, 6]]

Out[12]= {{0, 1, 0, 0, 0, 1}, {1, 0, 0, 0, 0, 1}, {1, 0, 0, 0, 1, 0}}

In[13]:= Hilb[M, 6]
```

Out[13]= $\frac{1 + 2 q}{(-1 + q)^4}$

```
In[14]:= H = {}; For[i = 1, i ≤ 4, i++,
          f = Sum[Random[Integer, 1000] * x[j], {j, 1, 6}]; H = Append[H, f]]

In[15]:= H

Out[15]= {658 x[1] + 860 x[2] + 662 x[3] + 749 x[4] + 609 x[5] + 992 x[6],
          91 x[1] + 290 x[2] + 861 x[3] + 42 x[4] + 69 x[5] + 333 x[6],
          303 x[1] + 14 x[2] + 619 x[3] + 607 x[4] + 427 x[5] + 881 x[6],
          12 x[1] + 968 x[2] + 388 x[3] + 52 x[4] + 520 x[5] + 478 x[6]}

In[16]:= G = GroebnerBasis[Union[G, H], Array[x, 6]]

Out[16]= {x[6]^2, x[5] x[6], x[5]^2, 2218664907 x[4] + 2895665445 x[5] + 3667892510 x[6],
          -17749319256 x[3] + 436701312 x[5] - 5724775277 x[6],
          11832879504 x[2] + 5816356332 x[5] + 3420392285 x[6],
          1479109938 x[1] - 1742084007 x[5] - 1592299202 x[6]}

In[18]:= M = MkLeads[G, Array[x, 6]]

Out[18]= {{0, 0, 0, 0, 0, 2}, {0, 0, 0, 0, 1, 1}, {0, 0, 0, 0, 2, 0},
          {0, 0, 0, 1, 0, 0}, {0, 0, 1, 0, 0, 0}, {0, 1, 0, 0, 0, 0}, {1, 0, 0, 0, 0, 0}}

In[19]:= Hilb[M, 6]

Out[19]= 1 + 2 q
```