Extra Problems 3/4/09

1. Clearly $x^2 - x + 41$ is not a prime for $x = 41$. Show that it is a prime for $1 \leq x \leq 40$.

2. Show that $\begin{pmatrix} 2n \\ n \end{pmatrix} \geq \frac{2^{2n}}{2n}$ for $n \geq 4$. Use this lower bound in place of the one we used (2^n) to get a better lower bound for $\pi(x)$.

3. Let p_n be the nth prime. Prove that $p_n \leq 2^n$.