

Another practice Quiz problem:

Consider the path $\sigma(t) = (t, 1 - t^2, \frac{2t^3}{3})$ for $0 \leq t \leq 1$.

a) Reparametrize this path to have the opposite orientation.

b) Calculate $\int_{\sigma} (xy + yz) ds$.

c) Calculate $\int_{\sigma} -z dx + x dz$.

Solution:

a) If we reparametrize $[0, 1]$ using $t(u) = 1 - u$ for $0 \leq u \leq 1$ and substitute getting $\sigma(t(u))$ then we have

$$(1 - u, 2u - u^2, \frac{2}{3} - 2u + 2u^2 - \frac{2}{3}u^3)$$

is a parametrization with the opposite orientation since $\frac{d}{du}t = \frac{d}{du}(1 - u) = -1$.

b) $\int_{\sigma} (xy + yz) ds = \int_0^1 (t(1 - t^2) + (1 - t^2)\frac{2}{3}t^3)\sqrt{1 + 4t^2 + 4t^4} dt =$

$$\int_0^1 (t(1 - t^2) + \frac{2}{3}(1 - t^2)t^3)(1 + 2t^2) dt = \frac{19}{36}.$$

c) $\int_{\sigma} -z dx + x dz = \int_0^1 -\frac{2}{3}t^3 dt + td(\frac{2t^3}{3}) = \int_0^1 (-\frac{2}{3}t^3 + 2t^3) dt = \frac{1}{3}.$