$x \in \text{V}(M), \ x_p \neq 0$.

**Lemma.** There exists $(U, \Phi)$ a chart for $M$ with $p \in U$ and $X|_U = \frac{\partial}{\partial x_i}$, where $x_1, \ldots, x_m$ are the corresponding local coordinates in $U$.

**Proof.** Let $(V, \Phi)$ be a cusick chart (i.e. $\Phi(V) = C^\omega_0$, $\Phi(p) = 0$) such that if $y_1, \ldots, y_m$ are the corresponding local coordinates, then...
Then $x_0 = \sum a_i \frac{\partial}{\partial y_{i_0}}$ with $a_{i_0} \neq 0$.

and also $\Phi_t^X : V \to M$ exists for $|t| < \varepsilon, (\varepsilon > 0)$

$$H(t, z) = \Phi(t, \Phi^{-1}(z))$$

$z \in C_a^m, \quad t \in C_{\alpha}^m$

$G(t, z) = t \lambda(t, \langle z, 0 \rangle).$

$C_t : (-\varepsilon, \varepsilon) \times C_{\alpha}^{m-1} \to \mathbb{R}^m.$

$\frac{dG_{t, 0, 0}}{dt} (2) = d\Phi(p(x_0)), \quad dG_{t, 0, 0} (e_j) = 0 \quad j = 1, \ldots, m-1.$ (4) $\Phi_0 = I \quad e_j$
Hence $dC_{\nu,0}$ is bijective.

If $b > 0$, $b < \epsilon$, $b < a$ and $R_t$

$G : C_b^m \rightarrow G(C_b^m)$ open is a $\times$,

diffeomorphism. \[ W \]

\((W,G)\) has property that \[ \frac{\partial^2}{\partial x_1} = d\Phi^{-1}(X) \] pull back

+ $M$. 


Want to follow:

1. \( p \mapsto V_p \) with \( V_p \subseteq T_p(M) \) a vector subspace.
2. Assume \( \dim V_p = n \) for all \( p \in M \).
3. For \( p \in M \) \( \exists \), \( U \) open, \( p \in U \) and \( x_1, \ldots, x_n \in \text{Vect}(U) \) such that \( V_q = \text{span}_k (x_{1q}, \ldots, x_{nq}) \) for all \( q \in U \).

We call this an \( n \)-dimensional sub-bundle of \( M \) tangent bundle.
If \( p \in M \) does there exist an \( n \)-dimensional submanifold \( N \), with \( p \in N \) and
\[
d_i(q^{-1} N) = V_q \quad \text{for all} \quad q \in N. \quad \left( \Psi \right)_w)
\]

Assume that \( N \) exists for \( p \).

Then \( \exists (U, \Psi) \) cubic chart at \( p \).
\[
\Psi(U) = C^n_0
\]
\( \Psi^{-1}(C^n_0) \) such that \( \mathbb{R}^n \times \delta \) to the chart for \( N \).
\[
v \in V_q, \quad q \in W, \quad v = \sum_{i=1}^{n} a_i \frac{\partial}{\partial y_i} \bigg|_q
\]
If $X_1, \ldots, X_n$ are as in (3) for $U$ (possibly having shrink $U$)

Then $x_i \bigg| \bigg. = \sum_{j=1}^{n} a_{ij} \frac{2}{\partial y_j}.$

Now calculate $\left[ X_i, X_j \right]$

\[ x = y \sum_{l \leq n} a_{ij} \left( \frac{\partial}{\partial x_j} a_{lk} \right) \frac{\partial}{\partial x_l} - \sum_{l \leq n} a_{kl} \left( \frac{\partial}{\partial x_i} a_{ji} \right) \frac{\partial}{\partial x_l} \]
$[X_i, X_j]_g \in V_g$.

Same argument if $X, Y \in \text{Vect}(M)$ are such that $X_g, Y_g \in V_g$ for all $g \in G$. Then $[X_i, Y_j]_g \in V_g$ for all $g \in G$. This condition is called integrability, i.e., $V_g$ is called an integrable subbundle of $T(M)$ if and only if satisfied.

(Warnen called this a "distribution")
A submanifold $N$ of $M$ is called an integral submanifold for $V$ if for each $p \in N$, $T_p(N) = V_p$.

**Theorem.** Given $V$ an integrable subbundle of $T(M)$ of dim $n$, and $p \in M$ there exists a cubic chart $(U, \Phi)$ for $M$ at $p$ such that the slice

$$y_{n+1}(q) = 0, \ldots, y_m(q) = 0 \text{ in } U$$
is an integral submanifold for \( V \).

Make \( M \) into an integral submanifold of \( V \).

On \( M \) we put a new topology defined as follows: \( \pi_{\text{dim}} \) through \( p \).

If \( p \in M \) let \( (U, \phi) \) be a cubic chart for \( p \) in \( M \). We take \( S_{\text{dim}} \) for a number of the topology at \( p \) in the upper subsets of the slice. We use all cubic slices through \( p \).
We meet prove that $M$ with this proposed basis
is a topology in a topological space.

(1) $\emptyset, M$ open

(2) A union of open sets is open.

(3) Intersection of a finite number of open sets is open.