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\(P(A|R)\) probability that \(A\) occurs and not \(R\).
\(D(A, R)\) probability that exactly 1 of \(A\) and \(R\) occur.

Bell's Inequality

\[P(A|R) + P(R|A) \geq P(A|S)\]

\[\Rightarrow P(A/R_1) + P(R_1/R_2) + \ldots + P(R_{n-1}/R_n) + P(R_n/S) \geq P(A|S)\]

\[A \rightarrow B\]

\[\Psi = \frac{1}{\sqrt{2}} (|100\rangle + |111\rangle)\]

\[= \frac{1}{\sqrt{2}} (|100\rangle + |\pi/2 - \theta, \pi/2, \theta\rangle)\]

Since \((P(\theta) \otimes P(\theta)) \Psi = \Psi\) \(P(\theta)\) notation removed \(\theta\).

A: particle with polarization \(\theta\), probability \(P(\theta)\)

we have a filter \(S\) polarizes to \(\phi\), \(P(S)\) the probability

\[
\langle (\cos \theta, \sin \theta) | (\cos \phi, \sin \phi) \rangle^2
\]

\[= (\cos \theta \cos \phi + \sin \theta \sin \phi)^2\]

\[= (\cos (\theta - \phi))^2\]

Thus

\[P(S|A) = P(\theta) [1 - \cos (\theta - \phi)^2]\]

\[= P(\theta) \sin (\theta - \phi)^2\]

A: Photon

\(Q, S\): filters of add an angle of \(\phi\).
\(Q_A, S_A\): insances of these filters. Similarly for \(B\)

\(R \rightarrow T\)
\[ P(\Omega A / R B) + P(\Omega B / S A) + P(\Omega A / T B) - P(\Omega A / T B) \]

want to show this is negative.

\[ = P(\Omega A / R B) + P(\Omega B / S B) + P(\Omega B / T B) - P(\Omega B / T B) \]

\[ = \frac{1}{2} (3 \sin^2 \phi - \sin^3 \phi) =: f(\phi) \]

(since all polarizations are equally probable and are 1/2)

if \( \phi = \frac{\pi}{6} \), get \( -\frac{1}{8} \)

\[ f(\phi) = \frac{1}{2} (3 \phi^2 + 0(\phi^3) - 9 \phi^2 + 0(\phi^3)) \]

\[ = \frac{1}{2} (3 \cdot 9 \phi^2 + 0(\phi^3)) \]

\[ \Rightarrow \text{get } \frac{1}{8} (1 - \Delta^2) \]

Exercise: \( \sin(\Delta) = \frac{e^{i\phi} - e^{-i\phi}}{2i} \)

Hint: use

Observation:

1. Classical probability is violated in quantum mechanics (Entanglement is critical)

2. Locality is violated.

\[ \text{EPR} \]

\[ \text{GHZ} \]

What follows is a method of using the GHZ state to demonstrate a quantum mechanical effect.

We look at 3 independent spin-1/2 states yielding the state

\[ \frac{1}{\sqrt{2}} (|000\rangle + |111\rangle) = \text{GHZ} \]

in 3-qubits.
We consider in qubit that
two observables $X$ and $Y$. Take values $\pm 1$

$$X^2 = Y^2 = I$$

Suppose observation of $3 \otimes 3 \otimes Y_3$ eigenvalue $-1$

Then

$$X_1 \otimes X_2 \otimes X_3$$

must be

$$X_1 \otimes X_2 \otimes X_3$$

with two factors having

a $Y$ in always $-1$.

Then

$$\begin{pmatrix} X_1 & Y_1 & Y_1 \end{pmatrix} \begin{pmatrix} X_1 & X_2 & X_3 \end{pmatrix} \begin{pmatrix} X_1 & X_2 & X_3 \end{pmatrix} \rightarrow -1$$

Quantum measurement:

$$X = \sigma_1, \quad Y = \sigma_2$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} \sigma_1 \sigma_1 = 11 \rangle, \quad \sigma_1 \sigma_1 = 10 \rangle \\
\sigma_2 \sigma_2 = 11 \rangle, \quad \sigma_2 \sigma_2 = -10 \rangle$$

$$X_1 Y_2 Y_3 |GHZ\rangle = (\sigma_1 \otimes \sigma_2 \otimes \sigma_3) (|10\rangle \otimes |01\rangle + |11\rangle \otimes |10\rangle) / \sqrt{2}$$

Satisfies the condition

$$-GHZ.$$ The three give

$$X_1 Y_2 Y_3 \quad Y_1 Y_2 Y_3 \quad Y_1 Y_2 Y_3 \quad GHZ = -GHZ$$

This gives

$$X_1 X_2 X_3 \quad GHZ = GHZ$$

Classically should get $-GHZ$
Exercise: Show \( \neq x, y \) for \( |000\rangle \)
\[
\frac{1}{\sqrt{3}} (|100\rangle + |010\rangle + |100\rangle)
\]
Can you find \( x, y \) for this?