Note Title 2/27/2009

$$\sum_{i=0}^{m} f^{(i)}(x) = \sum_{i=0}^{m} \sum_{j=0}^{m} \frac{d^i}{dx^i} c_j x^j$$

$$= \sum_{i=0}^{m} \sum_{j=i}^{m} \frac{d^i}{dx^i} c_j x^j$$

$$= \sum_{j=0}^{m} \sum_{i=0}^{j} c_j \frac{j!}{(j-i)!} x^{j-i}$$

$$= \sum_{j=0}^{m} c_j j! \sum_{i=0}^{j} \frac{x^{j-i}}{(j-i)!}$$

$$= \sum_{j=0}^{m} c_j \, j! \sum_{k=0}^{j} \frac{x^k}{k!}$$

$$H_f(x) = e^x \sum_{j=0}^{m} f^{(j)}(0) - \sum_{j=0}^{m} f^{(j)}(x)$$

$$\sum_{j=0}^{m} f^{(j)}(0) = \sum_{j=0}^{m} c_j \, j!$$

$$H_f(x) = \sum_{j=0}^{m} c_j \, j! \sum_{k=j+1}^{\infty} \frac{x^k}{k!}$$

$$= \sum_{j=0}^{m} c_j x^j \cdot \sum_{k=j+1}^{\infty} j! \frac{x^{k-j}}{k!}$$

$$\frac{j!}{k!} = \frac{1}{k(k-1)\cdots(j+1)} \leq \frac{1}{(k-j)\cdot(k-j-1)\cdots 1} = \frac{1}{(k-j)!}$$

$$|H_f(x)| \leq \sum_{j=0}^{m} |c_j| \, |x|^j \cdot \sum_{k=j+1}^{\infty} |x|^{k-j} \frac{j!}{k!}$$

$$\leq \sum_{j=0}^{m} |c_j| \, |x|^j \cdot \sum_{k=j+1}^{\infty} |x|^{k-j} / (k-j)!$$

$$\leq e^{|x|} \cdot \sum_{j=0}^{m} |c_j| \, |x|^j .$$

Chebyshev worked 1800–1850

1859 Riemann 6 pages long.

Let $\pi(x)$ be the number of primes, p, with $1 < p \le x$, $x \geq 2$

There exist $C_1, C_2 > 0$ such that

$$C_1 \frac{x}{\log x} \le \pi(x) \le C_2 \frac{x}{\log x}$$

Gauss, Legendre, ... Conjectured: theorem.

$$\lim_{x \to \infty} \frac{\pi(x) \log x}{x} = 1.$$ Prime number

i.e. In C's theorem we could take

$c_2 = 1+\varepsilon$ for any $\varepsilon > 0$

$c_1 = 1-\delta$ " " $\delta > 0$

1896 Hadamard and de la Vallee Poisson.

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$$\frac{\text{prime}(x)}{\log(\text{prime}(x))}$$

$x = 100, 1000,$
$10,000, \ldots$

$$\pi(\text{prime}(x)) = y_x$$ ~~if x~~ is a positive integer.