Note Title 3/13/2009

Final Exam: Friday CTR 201
11:30 – 2:30

Kevin session Thursday
APM 7218 4:30–6:30

My office hours Thursday
APM 7456 1:00–3:00

Partitions: $i_1 \geq i_2 \geq \cdots \geq i_r \geq 1$ $\quad i_j \in \mathbb{Z}$

$i_1 + \cdots + i_r = n.$

$p(n)$ number of partitions of n.

$n=0$ $p(0)$

$n=1$ 1 $p(1) = 1$

$n=2$ 2, 1 1 $p(2) = 2$

$n=3$

3

2 1

1 1 1

$p(3)=3$

$p(10) = 42$

$p(4) = 5$

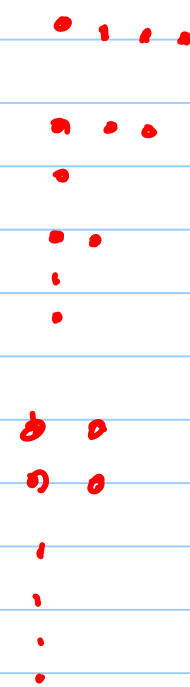
4

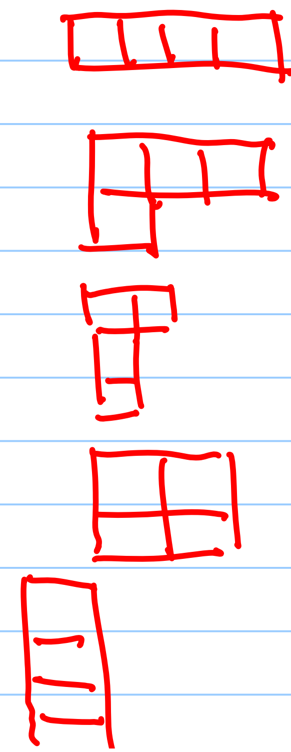
3 1

2 1 1

2 2

1 1 1 1





$$\frac{1}{\prod_{n=1}^{\infty}(1-q^n)} = \prod_{n=1}^{\infty}\left(\sum_{j=0}^{\infty} q^{jn}\right) = \sum_{n=0}^{\infty} p(n)\, q^n .$$

$$\underbrace{p(n) \sim e^{\pi\sqrt{n}}}$$

$$1 = \frac{\prod_{n=1}^{\infty}(1-q^n)}{\prod_{n=1}^{\infty}(1-q^n)} .$$

Is there a "nice" formula for $\prod_{n=1}^{\infty}(1-q^n)$?

$1-q \qquad \times\ 1-q^2$

$-q^2+q^3$

$1-q-q^2+q^3 \qquad \times\ 1-q^3$

$-q^3+q^4+q^5-q^6$

$1-q-q^2+q^4+q^5-q^6 \qquad \times\ 1-q^4$

$-q^4+q^5 \dots$

$1-q-q^2+2q^5+\dots$

$1-q-q^2+q^5+q^7-q^{12}-q^{15}+$

Euler's formula:

$$\prod_{n=1}^{\infty}(1-q^n) = 1 + \sum_{m=1}^{\infty}(-1)^m\left(q^{\frac{m(3m-1)}{2}} + q^{\frac{m(3m+1)}{2}}\right)$$

$$1 = \prod_{n=1}^{\infty}(1-q^n)\cdot\sum_{n=0}^{\infty}p(n)q^n.$$

$$\rightarrow \sum_{n=0}^{\infty}p(n)q^n = \sum_{\substack{n=1\\m=1}}^{\infty}p(n)q^n\cdot(-1)^m\left(q^{\frac{m(3m-1)}{2}} + q^{\frac{m(3m+1)}{2}}\right)$$

$$p(n) = \sum_{m \geq 1} (-1)^{m+1} \left(p\left(n - \frac{m(3m-1)}{2}\right) + p\left(n - \frac{m(3m+1)}{2}\right) \right),$$

$$p(0) = 1 \quad , \quad p(j) = 0 \text{ if } j < 0$$

Euler's <u>recursion</u>.

Let $p_d(n)$ be the number of partitions $i_1 > i_2 > \cdots > i_r \geq 1$

$\sum i_j = n$. (d for distinct)

$p_{d,e}(n)$ number of such with r even

$p_{d,o}(n)$ number with r odd

$$\prod_{n=1}^{\infty}(1-q^n) = \sum_{n=0}^{\infty}\left(p_{d,e}(n)-p_{d,o}(n)\right)q^n$$

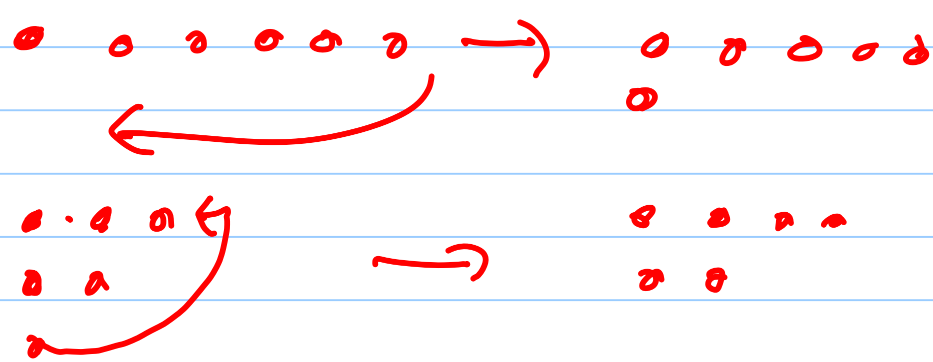
We claim $p_{d,e}(n) - p_{d,o}(n) \in \{0,1,-1\}$

6 o

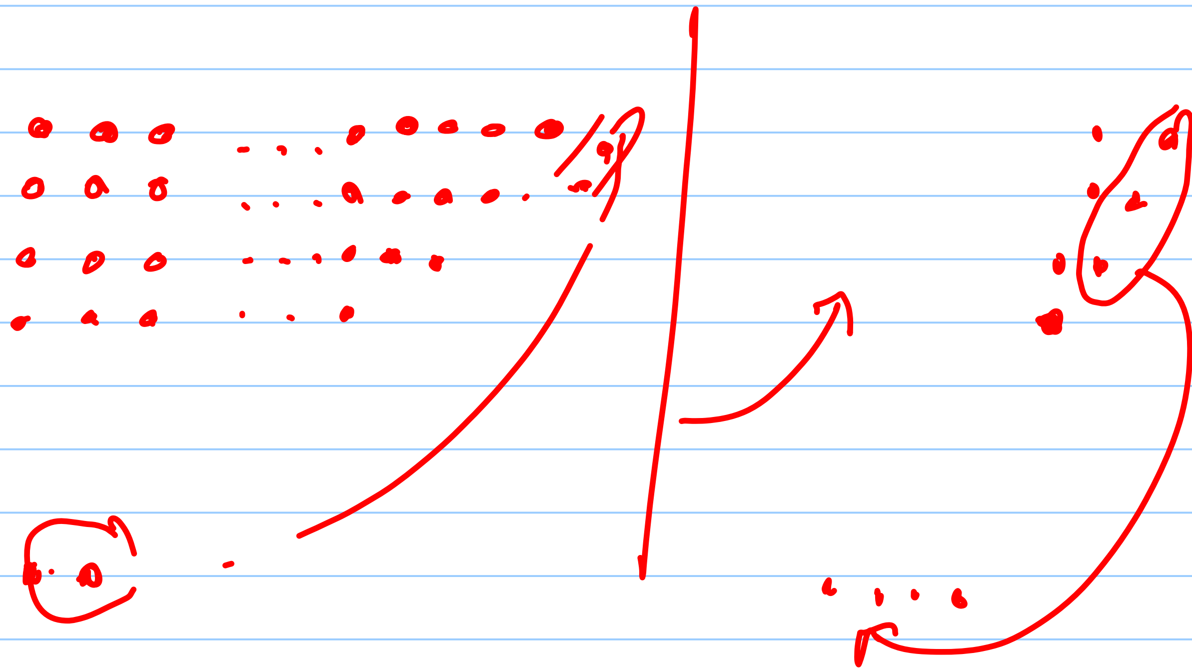
5 1 e

4 2 e

3 2 1 o



$i_1 > i_2 > \cdots > i_r \geq 1$



The way it gets proved.