p.252 10. Here we have

$$\begin{aligned}\frac{\partial f}{\partial x} &= 2e^{2x+y}, \frac{\partial f}{\partial y} = e^{2x+y}, \\ \frac{\partial^2 f}{\partial x^2}(x,y) &= 4e^{2x+y}, \frac{\partial^2 f}{\partial x \partial y}(x,y) = 2e^{2x+y}, \frac{\partial^2 f}{\partial y^2}(x,y) = e^{2x+y}.\end{aligned}$$

Thus the Taylor polynomial at $(0,0)$ is

$$p_1(x,y) = 1 + 2x + y.$$

If we wish to estimate how close this approximation is in some range of values neat $(0,0)$ let us assume we are interested in the points in the square $|x| < 0.1, |y| < 0.1$ then in this range we have

$$\left|\frac{\partial^2 f}{\partial x^2}(x,y)\right| = 4e^{2x+y} \leq 4e^{.3} \leq 4 \times 1.35 = 5.4.$$

Here we have used $e^{.3} \leq 1.35$ we also have

$$\left|\frac{\partial^2 f}{\partial x \partial y}(x,y)\right| \leq 2.7, \left|\frac{\partial^2 f}{\partial y^2}(x,y)\right| \leq 1.35.$$

Thus the Lagrange remainder in this range is at most

$$\frac{1}{2}\left(5.4|x|^2 + 5.4|x||y| + 1.35|y|^2\right).$$

in absolute value. For example if $(x,y) = (.01, .05)$ then the value of the Taylor polynomial is $1 + .02 + .05 = 1.07$ and the error is at most

$$\frac{1}{2}(5.4(.001) + 5.4(.005) + 1.35(.025)) = 0.0033.$$