

Practice Quizzes II

Practice Quiz 1:

Let $R = \{(x, y) \mid |x - 1| < 1, |y - 1| < 1\}$. Suppose that f is a real valued function on R that has continuous first and second partial derivatives. Assume that $f(1, 1) = -1$ and

$$\begin{aligned}\frac{\partial f}{\partial x}(1, 1) &= 1, \\ \frac{\partial f}{\partial y}(1, 1) &= 2.\end{aligned}$$

Also assume that if (x, y) are in R then

$$\left| \frac{\partial^2 f}{\partial x^2}(x, y) \right| \leq 1, \left| \frac{\partial^2 f}{\partial x \partial y}(x, y) \right| \leq 1, \left| \frac{\partial^2 f}{\partial y^2}(x, y) \right| \leq 1.$$

Calculate the first Taylor polynomial, p_1 , of f at $(1, 1)$. Use this to calculate an approximate value for $f(1.1, 1.1)$ and give an estimate for the accuracy of this approximation.

Practice Quiz 2:

Define $\beta(x)$ to be given by $\beta(x) = \sin(x)$ for $0 \leq x \leq \frac{\pi}{4}$ and $\beta(x) = \cos(x)$ for $\frac{\pi}{4} \leq x \leq \frac{\pi}{2}$. Draw the domain

$$D = \{(x, y) \mid 0 \leq x \leq \frac{\pi}{2}, 0 \leq y \leq \beta(x)\}.$$

Calculate

$$\iint_D \sin x \, dA.$$

Hint: $\sin^2 x = \frac{1 - \cos 2x}{2}$.