Practice Exam Math 109

10 points 1. Give the contrapositive of the statement:
If n is divisible by four then n is even.

15 points 2. Find the truth table of the following statements. Which of the statements are equivalent?
(a) not (P and Q).
(b) not (P or (not Q))
(c) (not P) or (not Q)
(d) not($P \Longrightarrow Q$)

20 points 3. Assume that we have a set A with addition $a+b$ for $a, b \in A$ satisfying the following four rules(axioms):
(i) There exists $0 \in A$ such that $a + 0 = a$ for all $a \in A$.
(ii) If $a, b \in A$ then $a + b = b + a$ (commutative rule).
(iii) If $a \in A$ there exists $b \in A$ such that $a + b = 0$.
(iv) If $a, b, c \in A$ then $a + (b + c) = (a + b) + c$ (associative rule).
Prove the following assertions using these rules.
(a) If $a \in A$ is such that there exists $b \in R$ such that $a+b = b$ then $a = 0$.
(b) Let $a \in A$. If $b, c \in A$ are such that $a + b = a + c$ then $b = c$.

15 points 4. Prove for all elements of $\mathbb{Z}^+ = \{1, 2, 3, ...\}$

$$\sum_{m=1}^{n} m^2 = \frac{n(n+1)(2n+1)}{6}.$$

20 points 5.Define the sequence A_n for $n \in \mathbb{Z}^+$ by

$$A_1 = 1, A_2 = 3, A_{n+1} = A_n + A_{n-1}, n \geq 2.$$

Let F_n be the Fibonacci sequence (defined by

$$F_1 = 1, F_2 = 1, F_{n+1} = F_n + F_{n-1}, n \geq 2.)$$

(a) Prove that $A_n = F_n + 2F_{n-1}$ for $n \geq 2$.
(b) Suppose that m is a positive integer and B_n is defined by

$$B_1 = 1, B_2 = m, B_{n+1} = B_n + B_{n-1}, n \geq 2.$$

What is the relationship between B_n and F_n?

20 points 6. For each of the following functions indicate if it is injective (one to one) or surjective (onto):

(a) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^4$.

(b) $f : \mathbb{R}^{\geq} \to \mathbb{R}$ defined by $f(x) = x^4$. (Recall $\mathbb{R}^{\geq} = \{x \in \mathbb{R} | x \geq 0\}$.)

(c) $f : \mathbb{R}^{+} \to \{x \in \mathbb{R} | 0 < x < 1\}$ defined by $f(x) = \frac{1}{1+x}$.