1. Let \( \mathfrak{g} \) be a Lie subalgebra of \( M_n(\mathbb{C}) \) such that every \( x \in \mathfrak{g} \) is diagonalizable as an element in \( M_n(\mathbb{C}) \). Show that if \( x, y \in M_n(\mathbb{C}) \) then \([x, y] = 0\). Hint: If \( x \in M_n(\mathbb{C}) \) is diagonalizable then \( \text{ad}(x) \) is diagonalizable on any subspace of \( M_n(\mathbb{C}) \) that it leaves invariant. Recall \( \text{ad}(x)y = [x, y] \).

2. On \( GL(n, \mathbb{C}) \) we have two topologies the Zariski topology as a Zariski open subset of \( M_n(\mathbb{C}) \) and the standard topology coming from the topology of \( M_n(\mathbb{C}) \) thought of as homeomorphic with \( \mathbb{R}^{2n^2} \) using the Euclidean metric. Suppose that \( G \subset GL(n, \mathbb{C}) \) is a Zariski closed subgroup that is compact as a subspace of \( GL(n, \mathbb{C}) \) in the standard topology. Show that \( G \) is finite. Hint: Use the Jordan decomposition of elements and problem 1.