1. Let $\mathfrak{g}$ be a Lie subalgebra of $M_n(\mathbb{C})$ such that every $x \in \mathfrak{g}$ is diagonalizable as an element in $M_n(\mathbb{C})$. Show that if $x, y \in M_n(\mathbb{C})$ then $[x, y] = 0$. Hint: If $x \in M_n(\mathbb{C})$ is diagonalizable then $ad(x)$ is diagonalizable on any subspace of $M_n(\mathbb{C})$ that it leaves invariant. Recall $ad(x)y = [x, y]$.

2. On $GL(n, \mathbb{C})$ we have two topologies the Zariski topology as a Zariski open subset of $M_n(\mathbb{C})$ and the standard topology coming from the topology of $M_n(\mathbb{C})$ thought of as homeomorphic with $\mathbb{R}^{2n^2}$using the Euclidean metric. Suppose that $G \subset GL(n, \mathbb{C})$ is a Zariski closed subgroup that is compact as a subspace of $GL(n, \mathbb{C})$ in the standard topology. Show that G is finite. Hint: Use the Jordan decomposition of elements and problem 1.