Quiz 1 Math 20E Fall,2004

Lines L_1 and L_2 are given parametrically with L_1 the set of all points whose position vectors are

$$t(1,1,1)+(0,1,1)$$

and L_2 those with position vector

$$t(2,0,1)+(1,1,0).$$

Find the distance between the lines L_1 and L_2.

Hint: Find parallel planes P_1 and P_2 such that P_1 contains L_1 and P_2 contains L_2. Calculate the distance between P_1 and P_2.

Solution: P_1 and P_2 must have a common normal vector which is perpendicular to direction vectors of each of the lines. Thus we can take the normal

$$\begin{aligned}\mathbf{n} &= (1,1,1)\times(2,0,1) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{vmatrix} \\ &= \mathbf{i}+\mathbf{j}-2\mathbf{k} = (1,1,-2).\end{aligned}$$

The planes can be described as follows P_1 is

$$x+y-2z=-1$$

and P_2 is

$$x+y-2z=2.$$

We have seen that the distance between these two planes is given by

$$\frac{|\mathbf{n}\cdot(\mathbf{a}-\mathbf{b})|}{\|\mathbf{n}\|}$$

with $\mathbf{a}$ a position vector of a point in P_1 and $\mathbf{b}$ a position vector of a point in P_2. Here we can take $\mathbf{a}=(0,1,1)$ and $\mathbf{b}=(1,1,0)$. Now $\|\mathbf{n}\|=\sqrt{1+1+4}=\sqrt{6}$ and

$$|\mathbf{n}\cdot(\mathbf{a}-\mathbf{b})| = |(1,1,-2)\cdot(-1,0,1)| = |-1+0-2| = 3.$$

So the answer is $\frac{3}{\sqrt{6}}=\frac{\sqrt{6}}{2}$.