

# Solutions

MATH 10C Fall 2008

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Section: \_\_\_\_\_

Quiz 1

Date: October 10, 2008

Name: \_\_\_\_\_ Grade: \_\_\_\_\_

PID: \_\_\_\_\_

Show all work clearly and in order, and circle your final answers. Write neatly in the space provided. Justify your answers whenever possible. No calculators, books or cheat sheets. You may use one clean sheet of paper for calculations, that however will not be collected. You have 20 minutes to complete this 20 point quiz. Cheaters caught will get into real trouble!

Problem 1. (10 points) (Problem 23 / 10.1)

Suppose that  $g$  is a function which has continuous derivatives, and that  $g(5) = 3$ ,  $g'(5) = -2$ ,  $g''(5) = 1$ ,  $g'''(5) = -3$ .

What is the Taylor polynomial of degree 2 near 5 for the function  $g$ ? What is the Taylor polynomial of degree 3 near 5 for the function  $g$ ?

Solution:

$$\begin{aligned} P_2(x) &= g(5) + g'(5)(x-5) + \frac{g''(5)}{2}(x-5)^2 = \\ &= 3 + (-2)(x-5) + \frac{1}{2}(x-5)^2 = \\ &= \boxed{3 - 2(x-5) + \frac{1}{2}(x-5)^2} \end{aligned}$$

$$\begin{aligned} P_3(x) &= g(5) + g'(5)(x-5) + \frac{g''(5)}{2}(x-5)^2 + \frac{g'''(5)}{3!}(x-5)^3 = \\ &= 3 + (-2)(x-5) + \frac{1}{2}(x-5)^2 + \frac{-3}{6}(x-5)^3 = \\ &= \boxed{3 - 2(x-5) + \frac{1}{2}(x-5)^2 - \frac{1}{2}(x-5)^3} \end{aligned}$$

Problem 2. (10 points) (Problem 4 / 8.8)

The probability of a transistor failing between  $t = a$  months and  $t = b$  months is given by  $c \int_a^b e^{-ct} dt$ , for some constant  $c$ .

If the probability of failure within the first six (6) months is 10%, what is  $c$ ? (express your answer using natural logarithms).

Solution: We know

$$\int_0^6 c e^{-ct} dt = 10\% = 0.1$$

$$\begin{aligned} \text{Now } \int_0^6 c e^{-ct} dt &= -e^{-ct} \Big|_0^6 = -e^{-c6} - (-e^{-c \cdot 0}) = \\ &= 1 - e^{-6c} \end{aligned}$$

$$\text{Therefore } 1 - e^{-6c} = 0.1$$

$$e^{-6c} = 0.9$$

$$-6c = \ln 0.9$$

$$c = \frac{\ln 0.9}{-6} = -\frac{\ln 0.9}{6}$$