MATH 20B Fall 2008 **Section**:______
Instructor: Orest Bucicovschi

Quiz 2 **Name:** ______________________________**Grade**:______
Date: October 20, 2008 **PID:** ______________________________

Show all work clearly and in order, and circle your final answers. Write neatly in the space provided. Justify your answers whenever possible and sketch all relevant graphs. No calculators, books or cheat sheets. You may use one clean sheet of paper for calculations, that however will not be collected. You have 20 minutes to complete this 20 point quiz.

Problem 1.(Sect. 11.3 nr. 16) ((10 **points**)
Convert to an equation in rectangular coordinates

$$r = \frac{1}{\cos\theta - \sin\theta}$$

Hint: cross multiply

Solution:

From

$$r = \frac{1}{\cos\theta - \sin\theta}$$

by cross multiplication we get

$$r(\cos\theta - \sin\theta) = 1$$

and so

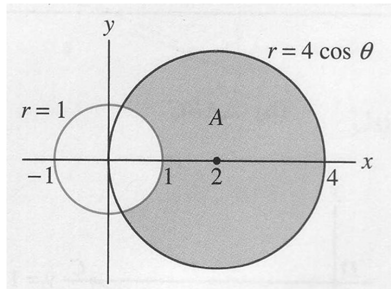
$$r\cos\theta - r\sin\theta = 1$$

The rectangular coordinates x, y are expressed in terms of the polar coordinates r, θ as

$$x = r\cos\theta$$
$$y = r\sin\theta$$

We get

$$\boxed{x - y = 1}$$



Problem 2. (Sect 11.4, nr. 13)(10 **points**)
Find the area of the shaded region A in the picture.

Hint: The area between two curves of equations in polar coordinates $r = f_1(\theta)$ and $r = f_2(\theta)$ is

$$Area = \frac{1}{2}\int_{\alpha}^{\beta}(f_1^2(\theta) - f_2^2(\theta))d\theta$$

What are α, β here?

Solution:

Take a common point of the two circles (there are two of them), with polar coordinates r, θ. Being on the first circle we have $r = 1$, being on the second circle we have $r = 4\cos\theta$. We get

$$1 = 4\cos\theta \quad \text{and so}$$
$$\cos\theta = \frac{1}{4}$$

This equation has two solutions in the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$, they are $\cos^{-1}(\frac{1}{4})$ and $-\cos^{-1}(\frac{1}{4})$. Therefore the lower point of intersection has angular coordinate $-\cos^{-1}(\frac{1}{4})$, the upper point of intersection has angular coordinate $\cos^{-1}(\frac{1}{4})$. We get the integration limits

$$\alpha = -\cos^{-1}(\frac{1}{4}) = -\beta$$
$$\beta = \cos^{-1}(\frac{1}{4})$$

We have

$$Area = \frac{1}{2}\int_{\alpha}^{\beta}(f_1^2(\theta) - f_2^2(\theta))d\theta = \frac{1}{2}\int_{-\beta}^{\beta}[(4\cos\theta)^2 - 1^2]d\theta =$$
$$= \frac{1}{2}\int_{-\beta}^{\beta}[16\cos^2\theta - 1]d\theta$$

Now $\int_{-\beta}^{\beta}[16\cos^2\theta - 1]d\theta = 7\theta + 4\sin 2\theta$. We get

$$Area = \frac{1}{2}[7\theta + 4\sin 2\theta]\ |_{-\beta}^{\beta} = 7\beta + 4\sin 2\beta$$

Now use: $\sin 2\beta = 2\sin\beta\cos\beta$ and $\cos\beta = \frac{1}{4}$, $\sin\beta = \sqrt{1-\cos^2\beta} = \frac{\sqrt{15}}{4}$ and we get

$$\boxed{Area = 7\cos^{-1}(\frac{1}{4}) + \frac{\sqrt{15}}{2}}$$