Math 104A, Number Theory, Fall 2002.
Summary of Lecture 2.

Mathematical induction.
We showed using the principle of mathematical induction that $n^2 < 2^n$ for $n \geq 5$.
We showed using the principle of complete induction that every positive integer greater than 1 is a product of primes.
We showed using the principle of mathematical induction that 5 divides $2^n + 3^n$ for every *odd* integer n.

Binomial Coefficient. The binomial coefficient $\binom{n}{r}$ (pronounced n choose r) is the coefficient of $a^r b^{n-r}$ in the expansion of $(a+b)^r$.

$$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^r b^{n-r}.$$

For example

$$(a+b)^3 = (a+b)(a+b)(a+b)$$
$$= aaa + aab + aba + abb + baa + bab + bba + bbb = a^3 + 3a^2b + 3ab^2 + b^3$$

$$\binom{3}{0} = 1, \qquad \binom{3}{1} = 3, \qquad \binom{3}{2} = 3, \qquad \binom{3}{3} = 1.$$

We can see that $\binom{n}{r}$ is the number of ways of choosing p objects out of a collection of n objects. To see that this is true, note that the objects in this case are the n brackets in the expression

$$(a+b)(a+b)\ldots(a+b).$$

For example, when $n = 3$, the objects are the 3 brackets

$$(a+b)(a+b)(a+b).$$

$\binom{3}{2}$ is the coefficient of a^2b in $(a+b)^2$. When we multiply out these brackets, how many terms give a^2b? We need to take a's from two of the brackets and a b from the remaining one, so this is the number of ways of choosing two out of three of the brackets to take a's from.