Math 104A, Number Theory, Fall 2002.
Summary of Lecture 7.

Definition. For $x \in \mathbb{R}$, $\lfloor x \rfloor$ is the largest integer less than or equal to x, that is the unique integer with $\lfloor x \rfloor \leq x < \lfloor x \rfloor + 1$. Furthermore $\lceil x \rceil$ is the smallest integer greater than or equal to x, that is the unique integer with $\lceil x \rceil - 1 < x \leq \lceil x \rceil$.

Examples. (a). $\lfloor 0.5 \rfloor = 0$, $\lceil 0.5 \rceil = 1$ $\lfloor -1.1 \rfloor = -2$, $\lceil -1.1 \rceil = -1$, $,\lfloor -5 \rfloor = -5$, $\lceil -5 \rceil = -5$.
(b). For $a, b \in \mathbb{R}$, the number of integers n with $a \leq n \leq b$ is $\lfloor b \rfloor - \lceil a \rceil + 1$.
(c). For $a \in \mathbb{R}$, $\lfloor a \rfloor$ is the number of integers n with $1 \leq n \leq a$.
We verified Lemma 2.1.13, in particular $\lfloor x + y \rfloor \geq \lfloor x \rfloor + \lfloor y \rfloor$.

Proposition 2.3.8. *If $p \leq n$, the exponent of p in the factorization of $n!$ is*

$$\left\lfloor \frac{n}{p} \right\rfloor + \left\lfloor \frac{n}{p^2} \right\rfloor + \left\lfloor \frac{n}{p^3} \right\rfloor + \ldots$$

We did the example of the case when 20!.
Now we have some consequences of the results from the last lecture.
Example. The numbers $5k + 3$ and $2k + 1$ are relatively prime. Indeed,

$$2(5k + 3) - 5(2k + 1) = 1.$$

We can also see this from $(a, b) = (a - bc, b)$ so $(5k + 3, 2k + 1) = (k + 2, 2k + 1) = (k + 2, -1) = 1$.
Example. Show that if a and b are integers, not both zero, the common divisors of a and b are the divisors of (a, b).
We showed Prop 2.3.4

Lemma. *If integers $p_1^{a_1} \ldots p_k^{a_k} | p_1^{b_1} \ldots p_k^{b_k}$ then $a_j \leq b_j$ for each j.*

Example If $a^2 | b^2$ then $a | b$.
Example. Write 570 and 123 as a product of primes and compute $(123, 570)$.
$570 = 2 \cdot 3 \cdot 5 \cdot 19$, $123 = 3 \cdot 41$ and $(570, 123) = 3$.
Example.

$$(2^6 \cdot 3 \cdot 5^2 \cdot 7^{41} \, , \, 2^{10} \cdot 3^{10} \cdot 5^{10} \cdot 7^{10}) = 2^6 \cdot 3 \cdot 5^2 \cdot 7^{10}.$$