Math 104B Winter 2003, Midterm 2: What's on the test?

5 Questions: 3 calculations, 2 proofs.

Here is a list of possible questions.

Define:

1. The Jacobi symbol $\left(\frac{a}{m}\right)$.
2. The kth convergent of the continued fraction $[a_0, a_1, \ldots]$.

Prove:

1. If the prime p divides a sum of two relatively prime squares then $p \equiv 1 \pmod 4$.
2. The product $(a^2+b^2)(c^2+d^2)$ can be expressed as a sum of two squares.
3. If a prime $q = a^2+b^2$ divides $n = x^2+y^2$ then n/q is a sum of two squares.
4. $p \equiv \pm q \pmod{4a}$ then $n(p) \equiv n(q) \pmod 2$.
5. Prove the quadratic reciprocity law. You may assume that if $p \equiv \pm q \pmod{4a}$ then $\left(\frac{a}{p}\right) = \left(\frac{a}{q}\right)$.
6. If $a_1, a_2, \ldots$ is a sequence of positive integers and $C_k = [a_0, a_1, \ldots, a_k]$, then $C_k = p_k/q_k$ where $(p_k, q_k) = 1$ and $p_k = a_k p_{k-1} + p_{k-2}$ and $q_k = a_k q_{k-1} + q_{k-2}$.

Find: (for given values of a, b, c, m, r, p, q)

1. Whether m is a sum of two squares.
2. Numbers x and y so that $x^2+y^2 = m$.
3. The primes which divide $x^2 - a$ for some x.
4. The value of the Jacobi symbol $\left(\frac{a}{m}\right)$.
5. The convergents of the continued fraction expansion of p/q and integers x, y solving $px - qy = 1$.
6. The continued fraction expansion of $a + b\sqrt{m}$.